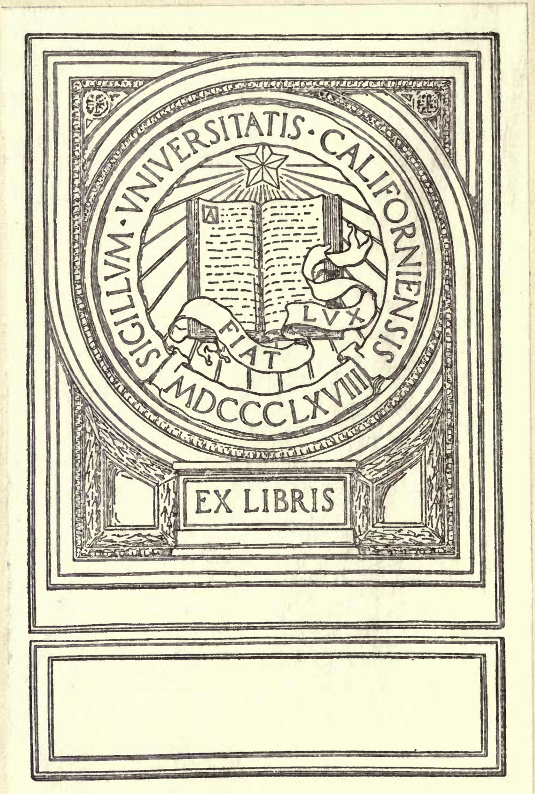
ELEMENTARY PRACTICAL MECHANIC

J. M. JAMESON

Ralph S. Minor



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BY

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PREFACE

THE accompanying text is an attempt to express practical mechanics as the science of the processes and structures of every-day life rather than as a series of more or less abstract mathematical demonstrations. It aims to convey not only a knowledge of facts and of fundamental theory, but also some training in ability to apply such knowledge; to equip the student who is to receive no further formal instruction in mechanics with practical information required for business or industrial life, and to establish a foundation and right point of view for the student who is to continue his study along applied lines, which shall enable him to advance without confusion or loss of time in acquiring new methods and a new vocabulary.

It is designed primarily as a text for elementary technical and manual training schools which find the usual texts in elementary mechanics too theoretical or too mathematical for their needs, and which require a more complete and practical course than that furnished by a text in general physics; also as an introductory text for engineering schools to be followed by the more applied courses in mechanism, and in the electrical and mechanical laboratories. The subjects of work, friction, and power transmission, and of power brake and dynamometer tests are discussed with considerable detail, and a chapter is

devoted to elasticity and to stress in materials. If properly supplemented by laboratory exercises, therefore, it is believed that satisfactory material will be found for a short course in applied mechanics.

It should, perhaps, be stated that the present text has grown out of a series of notes issued for several years in mimeograph form to the students in the School of Science and Technology, Pratt Institute. Nothing is included, therefore, which has not stood the test of several years' use with students in elementary technical courses. The time given to the subject at Pratt Institute is represented by one-half year with five lectures or recitations and six hours laboratory per week. The requirements in mathematics are limited to simultaneous and simple quadratic equations and the simplest trigonometric functions.

The order of presentation is that which has been found to give best results. General principles and definitions to form a ground work for laboratory exercises are introduced early, and the complete statement of theory, together with its applications, is then reached through laboratory and lecture work combined. The importance of carefully adapted laboratory exercises in teaching mechanics cannot be over emphasized. Numerous suggestions of models, etc., used in the laboratories at Pratt Institute are given in the figures illustrating this text.

In statics, as a rule, both the graphical and the analytical solutions are indicated. If desired, therefore, the use of trigonometric functions may be avoided without greatly modifying the ground covered in the text. Special effort has been made to present in a clear and usable form those portions of the text which deal with the mechanics of moving bodies, and to make the students' work in this subject something more than blind substitution of values in an uncomprehended formula. Repeated drill is pro-

vided in the application of the fundamental principle $f=ma$ to familiar instances, such as the starting and stopping of trains, the tension in hoisting ropes, etc., and of the corresponding expression for torque in rotary motion, to shafting, fly-wheels, etc. The conception of moment of inertia and radius of gyration is approached through the familiar ideas of action and reaction and of moment of force rather than through more abstract mathematical reasoning.

No apology is offered for the large amount of space devoted to problems. Too many problems in mechanics are scarcely possible. It is believed that the problems in this text will be found carefully selected and well distributed.

It is our practice to require the student to construct a simple, preliminary diagram of force conditions as a help to clear thinking, and as a basis for the final mathematical statement of his solution. Diagrams of this character have been inserted freely in the text, and an attempt has also been made to supply a number of cuts from photographs of actual commercial structures which, upon the specification of the necessary angles and dimensions by the instructor, shall supply material for valuable problems. In this connection, grateful acknowledgment is returned to Messrs. John Wiley & Sons, publishers, and to the respective authors, for permission to reproduce the following cuts: Fig. 91 taken from Sanborn's "Problems in Mechanics;" Fig. 103 from Merriman and Jacoby's "Roofs and Bridges;" Fig. 104 from Johnson, Bryan, and Turneaure's "Theory and Practice in the Designing of Modern Framed Structures;" and Fig. 155 from Carpenter's "Experimental Engineering;" also to the publishers of the *Railroad Gazette* for Fig. 102, and to the Brown Hoisting Machinery Company for numerous illustrations of various types of hoisting machinery.

In conclusion, the author wishes to express his very great indebtedness to Mr. Dana Pierce, Electrical Engineer for the Underwriters' Laboratories, jointly with whom, when associates at Pratt Institute, the original notes were written. Much of the plan and present form of the text is due to his valuable suggestions and assistance.

Grateful thanks are also extended to Mr. Arthur L. Williston, Director of the School of Science and Technology, Pratt Institute, for his encouragement and many suggestions in developing the course in mechanics; to my associates, Dr. Harrison H. Brown, Mr. William H. Timbie, and Mr. John A. Randall for helpful criticisms of the manuscript, and to Mr. Randall also for superintending the preparation of the illustrations and supplying the index.

J. M. Jameson.

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CONTENTS

CHAPTER IV

EQUILIBRIUM: THREE FORCES AT A POINT

CHAPTER V

PARALLEL FORCES. CENTER OF GRAVITY

CHAPTER VI

CONCURRENT FORCES APPLIED AT DIFFERENT POINTS. ALGEBRAIC CONDITIONS FOR EQUILIBRIUM

CHAPTER VII

FORCES IN SOME COMMON COMMERCIAL STRUCTURES

CHAPTER VIII

MOTION

CHAPTER IX

FORCES PRODUCING MOTION

CHAPTER X

WORK, POWER, ENERGY

CHAPTER XI

FRICTION. THE GENERAL LAWS OF MACHINES

ELEMENTARY PRACTICAL MECHANICS

CHAPTER I

FORCE

1. Mechanics.—Mechanics treats of forces and of the effects of forces. In the following pages we shall consider: 1st. Bodies acted upon by forces in such a way that they are compelled to continue *without change* in their existing condition of rest or motion, i.e., if at rest, are compelled to remain at rest, and if moving in a given direction or rotating at a given rate, are compelled to continue moving in the same direction or rotating at the same speed. 2d. Bodies compelled by forces to continually change their motion, i.e., made to move faster or slower or in a different direction. 3d. The effects of forces in changing the form of bodies by stretching, compressing, or bending them.

It is necessary, therefore, in beginning the study of mechanics, to have a clear conception of the meaning of the term *force*. The following discussion and illustrations are intended to make clear the significance of the term as used in practical mechanics:

2. Action and Reaction.—Suppose we have an ordinary spring balance lying upon a *perfectly smooth* horizontal

table. By "perfectly smooth" table we are to understand one along which the balance can be drawn without friction (a condition which of course can only be realized approximately for any actual table). If now we take hold of the ring, we may slide the balance steadily along the table, once started, with no sensation of effort; we will be *unable to pull upon it* unless we increase its speed. But suppose we pass the hook of the balance over a pin and then pull. We *may now apply a force* with the hand, the spring will be extended and the reading of the graduated balance will serve as a measure of this action tending to move the balance. The pin therefore provides a condition necessary before force can be applied, which, if we replace the pin by a second balance like the first, we find to consist of *an equal action in the opposite direction.* This opposing action is commonly spoken of as the *reaction.*

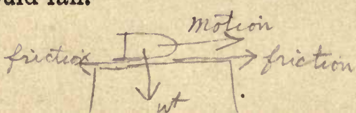
As other illustrations of the same idea, consider the following:

Example 1.—Suppose a rope with a boy pulling at each end. Here we have *two* actions, *equal* in amount and *opposite* in direction. *One boy cannot pull unless the other provides the opposing reaction.*

Example 2. —Suppose a rope tied to a post with a boy pulling at its other end. Here the post is taking the place of the second boy of Example 1, and is exerting the reaction equal and opposite to the pull which the boy is exerting.

Example 3.—Suppose a book resting upon a table. The earth is exerting a downward pull (gravity) upon the book, causing it to push against the table, and at the same time the table is exerting an upward pressure against the book which is equal and opposite to the force of gravity; if the table failed to exert this upward pressure the book would fall.

inter action
not reaction



Forces on Book
wt.

Reaction
Book pull on

Example 4.—Suppose a horse pulling a sled at a uniform Counter-act rate over level ground. Friction in this case is exerting an opposite and simultaneous reaction equal to the pull of the horse.

Example 5.—Suppose the horse pulling a sled, as in Example 4, increases his speed. *Two* reactions now oppose the pull applied to the sled. One, *friction*, opposes the slipping of the sled over the ground; the other, due to *inertia*, opposes increase of speed. These two together are equal and opposite to the pull exerted on the sled.

Many other illustrations might be given. In all, we should find the same fundamental conditions present, viz.:

1. *Two bodies* are always involved. What we term *force* is the "push, pull, attraction, repulsion, rubbing, pressure, etc., of *one body on another.*"

2. When one body, *A*, exerts force upon a second body, *B*, body *B* exerts an equal and opposite action on *A*, called the *reaction of B.* In speaking of the body *A* *alone*, we shall have frequent occasion later to include as among the "forces" applied to it, the reaction of the second body *B*.

3. The Effects of Force.—A force (i.e., the pressure, attraction, rubbing, etc., of some other body) applied to a given body may produce *one or both of two effects* upon that body:

First, it may *change the state of motion of the body* acted on; that is, may cause the body to move if at rest, or to move faster or slower or in a different direction if already in motion. The state of motion of a body under the action of no force (or of balanced forces) is either absolute rest or uniform motion.

Or, second, the force may *alter the shape of the body.* For example, the force may extend or compress a spring; it may deflect a beam; it may compress a column, as a

weight resting on a table compresses the legs of the table. As all known substances are compressible, and also may be elongated, some change of shape is always produced by an applied force, though perhaps the change may be so slight as to be negligible.

4. The Measurement of Forces.—The *quantity* of either of these *effects* on any given body is found to depend directly upon *the quantity of force* applied. Hence a force may be measured either:

(*a*) By the *velocity that it will impart to a given body in a given time,* or

(*b*) By the *amount of change of shape which it produces* in a given body.

Method (*a*) enables us to find the force required to start a body from rest, to stop a moving body, or to change the velocity of a body by a stated amount in a given length of time. (See Chapter IX.)

Method (*b*) is used in comparing two forces by determining the amount that each will stretch a spring, the angle through which each will twist a given rod or wire, etc. The spring balance is a familiar instrument for measuring forces in this manner. The elongations of the spring under the action of known forces, are marked on the scale of the instrument.

5. Definition of Force.—From the preceding discussion, we see that *force* is the *name given to the action which one body may exert upon another,* that the effects of such actions may be either change of motion or change of shape, and, finally, that actions between bodies must be mutual or in pairs, hence we may now state as our *definition of force:*

"*A force is one of a pair of equal, opposite, and simultaneous actions between two bodies by which the state of their motion is altered or a change in the form of the bodies themselves is effected.*"

6. Positive Actions and Reactions.—In many cases, either one of the mutual actions exerted between two bodies may be regarded as the *positive action* or the *force*, and the other one regarded as the *reaction* resisting the force. For instance, in Example 1, either one of the two boys may be regarded as exerting the *force*, and the other one as exerting the equal and opposite action spoken of in the definition, which in the subject of mechanics we are calling the *reaction*.

7. Passive Reactions.—There are, however, three instances in which we cannot assume either action to be the force. These are:

First, *Friction*. In Example 4, given above, it is impossible for us to suppose that the friction is exerting the force and that the horse is exerting the reaction. In such cases where friction is one of the equal, opposite, and simultaneous actions, it is necessary to regard the friction as the *reaction* and the other action as the positive action or force.

Second. Where the resistance of some body due to its *elasticity* or resistance to a change of shape is one of the actions, we cannot regard this as the positive action (unless it is tending to produce motion, as in the case of a spring under tension). In Example 3, for instance, it would not be proper to assume that the table was exerting the force and that gravity was exerting the reaction.

Third. Where the resistance of the body to change of velocity due to its own *inertia* * is one of the actions, we

---

* Inertia.—Inertia is to be regarded, not as a force, but as a *property of all matter*. "A body at rest will remain at rest, a body in motion will continue to move in the same direction and at the same speed, unless compelled to change its state of rest or motion by the application of an *external force*." The property of a body because of which force is necessary in order

must usually regard this as the reaction (except in certain instances where the velocity of a body is being destroyed, as in cases of impact between bodies).

These three types of reaction can be *developed by* the application of a positive action or force only. We could not logically think of their acting independently to develop an accompanying equal reaction. Nevertheless, where the positive action does exist, as, for illustration, in the case of gravity pull upon the book, Example 3, we may, when convenient, think of the *reaction of the table as an equal force upon the book*, and may thus speak of the book as under the action of *two forces*, without regard to their source: a downward force equal to the weight of the book, and an equal upward force, the reaction of the table. This idea of considering a given body apart from its surroundings and acted on by forces only, while purely imaginary, is important in the solution of problems in Mechanics. (See also Art. 16.)

8. Tension and Compression.—Attention has already been called to the fact that actions exist in pairs, and that we have force only as one of the mutual actions between two bodies; the student must not confuse this idea and interpret it as meaning that the force present is twice its proper value. It is *one* of the actions.

Where the actions are such as to tend to *pull the parts of the body apart*, the body is said to be *under tension*. This is the case, for example, of a rope tied at one end to a rigid support and sustaining a weight. Here gravity is exerting

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to produce change in its state of rest or motion, is spoken of as the *inertia* of the body. Whenever the velocity of a body is changing—increasing, decreasing, or changing direction—therefore, we must consider as *one of the reactions present, the resistance to change of motion* due to the body's inertia. (For further discussion of inertia reactions and force, see Chapter IX.)

a downward pull on the rope equal to W and the support is exerting an equal upward reaction, B. These external actions set up *internal actions* or *stress* in the rope and, at any section of the rope, the actions are opposite and directed *away* from the section, as a, b, Fig. 1. If, however, $W=20$ lbs., each action is 20 lbs. and we say, the *force of the tension* is 20 lbs., *not 40.*

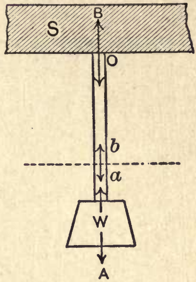


Fig. 1.

This is true, no matter what the sources of the action and reaction. Thus in the structure of Fig. 2, a jointed frame, ABC, pinned to turn freely at B, the weight at W tends to flatten the frame and cause ends A and C to separate. If slipping is prevented by a tie, AC,

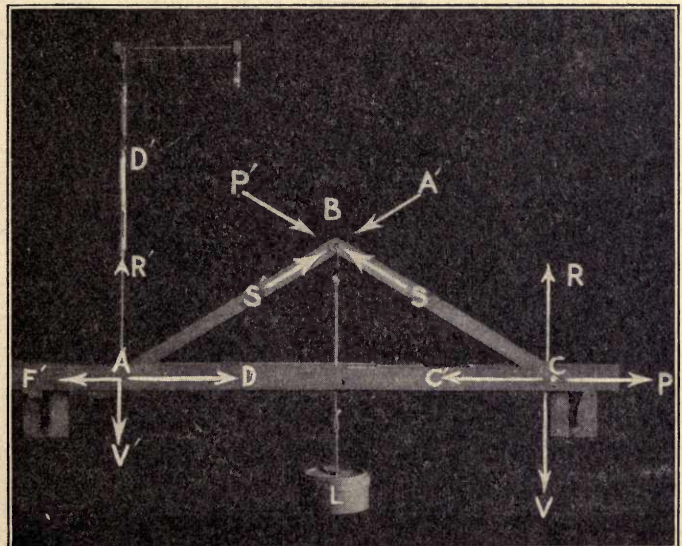


Fig. 2.—Laboratory Model Roof Truss.

and if C tends to move toward the right, with a force, P, of say 10 lbs., there must be an action, F', to the left at A of also 10 lbs. Either may be taken as the action, the other as the reaction. The tie, AC, is therefore under a tension of *10 lbs.* (*not* 20 lbs.), as will be shown by the spring balance at D.

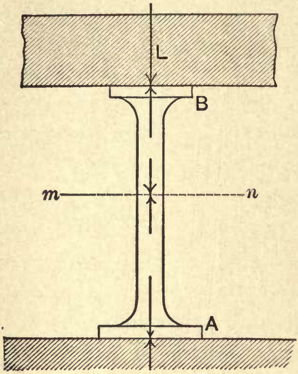


Fig. 3.

When the actions are such as to tend to *push the parts* of a body *together*, the body is said to be under *compression*. An illustration of this would be the column AB, Fig. 3, rigidly footed at A, and supporting a load L. At any section of the column, as mn, the actions are opposite and directed *toward* the section. If $L=100$ lbs., each action is 100 lbs., and the *force of compression is 100 lbs.*

9. Action and Reaction at any Point in a Structure.— Equal and opposite actions, or action and reaction, are present *at any point* in a structure that we may select for consideration. Thus in Fig. 3, action and reaction are equal at B: the action the pressure of the load L, and the equal reaction that due to the elasticity of the column which causes it to resist compression and push back equally against L. Action and reaction are equal at the section mn or any other section across the column. And action and reaction are also equal at A: the push of the foot of the column downward against the floor and the equal upward reaction of the floor, etc.

In the structure of Fig. 2, attention has already been called to the horizontal actions P and F', and the equal reactions supplied by the tension in the cord. If, how-

ever, we consider the vertical pressures V and V' at C and A, we have the equal vertical reactions R and R'.

Or, if we consider the pin connecting the members CB and AB at B, we have P' and A' as the pressure of the pin on BC and AB respectively, and S and S' the equal reactions of the timbers on the pin. S and S' are obviously *both holding the pin up,* and *pushing against one another,* or we have their combined effect as a vertical reaction M (Fig. 4), equal and opposite to load L, thus supporting the pin, and two equal and opposite actions N and N', either of which may be regarded as the action, the other as the reaction.

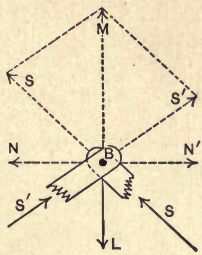


FIG. 4.

Many illustrations of the same character might be given. The first thing the student should do in the solution of any problem is to find the actions and reactions upon the body under consideration. These once properly arranged, subsequent operations become comparatively simple and mechanical computations.

PROBLEMS

Point out *the actions and reactions* at any selected point in the following:

1. A weight suspended from a hook by means of a cord.

2. A man lifting a heavy weight.

3. A coiled spring compressed and held in place by a detent.

4. A car being drawn at a uniform rate along a level track.

5. The car of Prob. 4, if drawn at a constantly increasing speed.

6. A block sliding down an inclined plank.

7. A barge being towed by a tug at uniform speed.

8. A car being drawn at uniform speed up an incline.

9. The car of Prob. 8, if its speed is increasing.

10. A moving ferry boat after the engines are suddenly stopped.

11. A mine car being lowered uniformly down a vertical shaft.

12. The car of Prob. 11, when lowered with increasing speed.

13. The lever *AC* of Fig. 12.

14. The simple truss, Fig. 41.

15. The roller and plane, Fig. 42.

16. The shears, Fig. 91.

17. The arch, Fig. 113.

18. The crane, Fig. 87.

19. The system of pulleys, Fig. 136, if *A* is moving downward at uniform speed.

CHAPTER II

GENERAL IDEAS OF FORCE. GRAPHICAL REPRESENTATION OF FORCE, MOTION, ETC.

The mechanics of bodies in motion will be treated in Chapters VIII and IX. A brief discussion of terms which we shall need to use frequently is given here.

10. Motion.—Motion may be defined as change of position. The change in position of a body is necessarily observed by comparing its successive positions with that of some other body for the time being regarded as fixed. *All motion is, therefore, relative.* A body may be in motion with respect to one body and at rest with respect to another. Thus, if a man walks down the aisle of a car he is in motion with respect to the car, but if he walks at the same rate and in the opposite direction to the motion of the car, he is at rest with respect to the track and ground. Again, a body may possess at the same time several different motions depending upon the points to which the motion of the body is referred. Thus, suppose a man is walking across a moving ship which in turn is headed straight across a stream having a current. He has one motion with respect to the ship, a different motion with respect to the water, and still a different motion with respect to the shore.

11. Translation and Rotation.—When a body moves in such a way that each point in the body passes over a path equal and parallel to the path of every other particle, the body is said to have motion of *pure translation.* The

motion of the piston rod of an engine, of a sled over ice, etc., are illustrations of motion of translation.

If, however, two points in the body are fixed and the body rotates about a line through these two called the *axis of rotation*, the motion is said to be one of *pure rotation.* A bicycle wheel, raised from the ground and set spinning on the shaft, the fly-wheel on a stationary engine, etc., are examples of bodies possessing a pure rotary motion.

Frequently a body may have both motion of translation and motion of rotation at the same time, as, for example, a wagon wheel when the wagon is drawn forward, a car wheel, etc. In general, a body may be moved from one position to any given second position in one of three ways: (*a*) By translation in a stated direction; (*b*) by rotation about a given axis; or (*c*) by translation a certain distance in a given line and then rotation through a definite angle about a selected axis.

12. Velocity.—By the *velocity* of a body we mean its *rate of motion* in any recognized direction. Velocity is therefore *measured by the amount of change* of position which occurs *in* one *unit of time.* Thus 30 miles per hour north, 20 ft./sec., etc., are expressions of velocity.

Since all motion is relative, velocity which fixes the rate of motion is also relative, and a body may have at the same time different velocities (either in amount or direction) with respect to different bodies to which the motion is referred.

13. Moment of Force.—One of the effects which may be produced by the application of force to a body has been stated to be change in the existing motion of such body (Art. 3). This change may be change in motion of translation or change in the rotation of the body, depending upon the conditions and upon the direction of the force

and the point at which it is applied. Thus suppose we pull upon the rope attached to a sled; we may cause the sled to move forward in the direction of the pull, the rate of change in its motion depending upon the amount of the applied pull.

A force applied to a shaft or pulley wheel, etc., may or may not tend to produce rotation. Thus consider the drive wheel in Fig. 5. When the connecting-rod is in the position *PR*, the force along it would tend merely to push the pin *P* against the axle *X*, since the force line *MN* of the connecting-rod passes through the axle. Any amount of force exerted by the rod under these conditions would not cause the wheel to rotate.

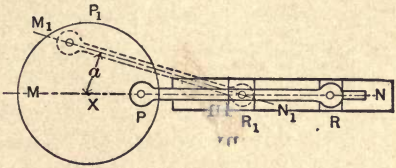


FIG. 5.

Let the pin, however, move to the position P_1, where the force line of the rod no longer passes through the axle, and it is easy to see that a force exerted along the rod would tend to produce rotation. The force is now said to have a *moment arm* (a), which is the *perpendicular distance* of its force line from the axle or axis of rotation. (A force *must* always possess this *arm* if it is to produce rotation.) The force is in this case said to have a moment about the shaft X as an axis.

Definition.—The tendency of a force to produce rotation about a given axis is called the moment of the force with respect to that axis.

14. Measure and Direction of Moment of a Force.— The *amount and direction* of the moment of a force depends upon the direction of the force and upon its distance from the axis. Thus suppose we have an apparatus such as shown in Fig. 6, in which *OQ* is a stiff rod balanced about a pin *P*,

and free to rotate in either direction about the pin. If now we hang a weight W, of 10 lbs. at M, 5 inches from the pin, this force will turn the rod in a clockwise direction about the pin. To prevent this we must have an *equal moment counter-clockwise,* and if this is furnished by a pull applied by a spring balance S, also attached 5 inches from P, we shall find this balance will read 10 lbs. (the same as W).

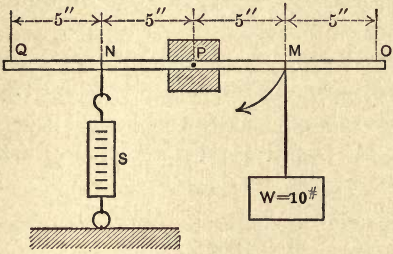


FIG. 6.

If W is now moved to O, 10 inches *or twice as far* from P, we shall find that the pull S' applied at N must be increased to 20 lbs. or *twice as much force.* And if we wish to balance W acting at O by an *equal force,* we must apply such force at Q, 10 inches or *an equal distance* from P.

In other words, we can balance a force by an *equal* force in an apparatus like this, if the two are applied at points equally distant from the pin; but *twice the force* is required to balance another force applied *twice as far* from the axis.

Two moments are thus equal *when the product of force $\times$ perpendicular distance from axis to force* is the same for both. Thus, by referring to Fig. 6, we have equal and oppositely directed moments under the following conditions:

$$(1) \quad W \times PM = S \times PN,$$

or

$$10 \times 5 = 10 \times 5.$$

$$(2) \quad W \times PO = S' \times PN,$$

or

$$10 \times 10 = 20 \times 5.$$

$$(3) \quad W \times PO = S \times PQ,$$

or

$$10 \times 10 = 10 \times 10.$$

The *measure of the moment of a force* is therefore the product of the force by the perpendicular distance from the axis to the action line of the force. Therefore,

Moment of force = force $\times$ moment arm.

Moment of force is also called *torque* when applied to wheels and pulleys.

The moment arm must always be measured along a direction *perpendicular to the direction of the force.* In Fig. 6 this is obviously along the stick. In Fig. 5, however, the moment arm of the force F exerted by the connecting-rod is the distance a where *a is perpendicular to the* direction of the force. F here *both* tends to turn the wheel and to pull it against the pin. Its rotation effect *alone* is $F \times a$, *not* $F \times$ distance $P'X$.

PROBLEMS

1. A railroad train passes a telegraph pole beside the track at rate of 45 miles per hour. What is:
   (*a*) Velocity of train with respect to pole?
   (*b*) Of pole with respect to train?
   How do these velocities compare in direction?

2. Two trains going at the rate of 15 miles per hour and 25 miles per hour respectively, in the same direction, pass each other at a station. What is:
   (*a*) Velocity of each with respect to the station?
   (*b*) Velocity of second with respect to first?
   (*c*) Velocity of first with respect to second?
   What is the direction of the motion in (*b*) compared with that of (*c*)?

3. Suppose trains in Prob. 2 to be going in *opposite* directions. What is:
   (*a*) Velocity of each with respect to station?
   (*b*) Velocity of second with respect to first?
   (*c*) Velocity of first with respect to second?

4. The velocity of light is 186,400 miles per second. How long does it take light to come from the sun distant 92,000,000 miles?

5. If the velocity of sound is 1100 ft. a second, how far distant is a gun whose report is heard $3\frac{1}{5}$ seconds after the flash is seen?

6. Express a velocity of 10 miles per hour in ft./sec.

7. Which is the greater velocity, 40 miles per hour, or 1800 ft./sec., and how much?

8. Through what distance per minute does a point on the rim of a fly-wheel move when the wheel makes 150 revolutions per minute? Diameter of wheel 5 ft.

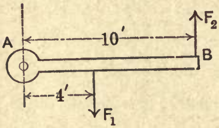


Fig. 7.

9. *AB* (Fig. 7) is a straight bar pinned at *A*. Force $F_1 = 50$ lbs., force $F_2 = 70$ lbs. What is moment of each about the pin?

10. Board *ABCD* (Fig. 8) is free to rotate about a pin at 0. Point out the moment arms of each of the indicated forces with respect to 0.

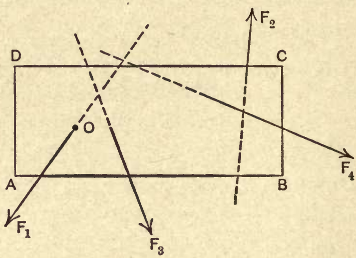


Fig. 8.

11. Suppose *AB* (Fig. 81) is a straight bar weighing 200 lbs., hinged to wall at *A*. If its weight acts at the center of the bar, what is moment of its weight about hinge? If bar is held from rotating by means of the rod *CB* what must be moment of pull in rod about hinge? What is the tension in the rod? $AB = 6$ ft., and $d = 2.2$ ft.

15. Use of Lines in Representing Velocities, Forces, etc.—A force is completely specified by stating: (*a*) Its amount; (*b*) its direction; (*c*) the point at which it is applied to the given body. Similarly, a motion is completely described by stating its amount, its direction, and

the original position of the moving body. Since a line may express these three attributes, i.e., may have a definite length, a definite direction, and may start from or end at a given point, a line may be used to represent a force, a motion, or any similar directed quantity. In representing a force by means of a line, the line is drawn to or from the point on the given body at which the specified force acts, in a direction corresponding to the direction of the force, and of a length to express the given force according to some previously determined and convenient scale, as, for example, 1 in. = 10 lbs., $\frac{1}{4}$ in. = 1 lb., etc. Velocities, motions, etc., may be represented by lines in a similar manner.

Thus lines *a*, *b*, and *c*, in Fig. 9, represent velocities of 15 ft./sec. West, 20 ft./sec. South, and 30 ft./sec. North-

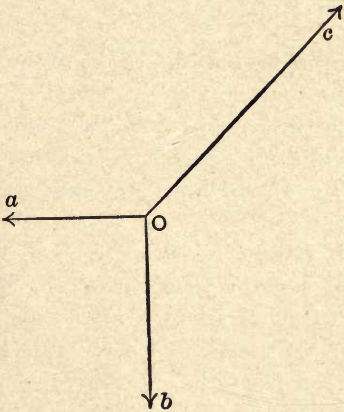


FIG. 9.

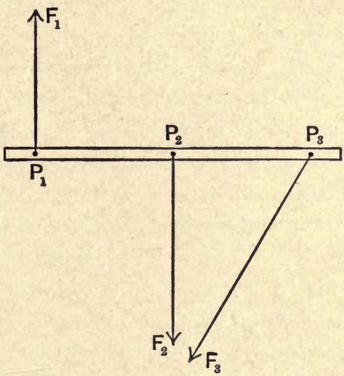


FIG. 10.

east, respectively, from the point 0, on the scale of $\frac{1}{4}$ in. = 5 ft./sec, i.e., line *a* is $\frac{3}{4}$ in. long, line *b* 1 in., and line *c* $1\frac{1}{2}$ in.

Lines F_1, F_2, and F_3 (Fig. 10) represent forces of 6, 8, and 10 lbs. respectively, applied to the rod *AB* at the

points P_1, P_2, and P_3, and acting in the directions indicated scale: $\frac{1}{8}$ in. = 1 lb. (i.e., F_1 $\frac{3}{4}$ in. long, F_2 1 in., F_3 $1\frac{1}{4}$ in.).

Fig. 11 represents graphically the forces acting upon a body weighing 100 lbs., being drawn at uniform speed up a plane AB inclined 30° to the horizontal by a pull P parallel to the plane, friction being equal to a pull of 30 lbs. The pull required is 80 lbs. R represents the reaction of the plane in supporting a part of the weight, W, of the body.

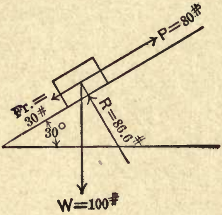


FIG. 11.

EXERCISES

INSTRUCTIONS FOR QUANTITATIVE GRAPHIC WORK

In all graphical work where actual values are to be shown, observe the following instructions:

(*a*) Have the pencils and instruments in good working order. The same care in making drawings will be exacted here as in regular drawing work. Broad lines and careless work cannot be accepted.

(*b*) Determine scale to which forces, velocities, etc., are to be drawn. This should be some *simple* ratio, as 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$ in. to a unit. Avoid thirds of an inch and other such inconvenient subdivisions. If metric units are used, do not mix decimals with common fractions.

(*c*) State the scale used. The drawing should in all cases be *as large as possible* with the page at your disposal. *One* diagram on a page.

(*d*) Arrange the figure so that it will come approximately in the center of the page, leaving a good margin for binding at the left.

(*e*) Put your name on every sheet and indicate clearly the number of the problem, but *do not* write out the problem on your drawing. If requested, submit the problem on another sheet.

(*f*) Place on lines their values expressed in *force* units, *velocity* units, etc. (*not* their actual lengths in inches, cm., etc.), and give value of all angles that are furnished or determined. Show directions by arrows.

(*g*) If the value of a line or angle is to be determined by your drawing, distinguish it clearly in some way from the lines and angles given by the problem, and state its value.

---

1. Represent by lines starting from a common point, and showing direction and amount:

   (*a*) Velocities of 60 ft./sec. South; 15 ft./sec. West; 30 miles per hour Southeast.

   (*b*) Forces of 30, 15, 8, and 10 lbs., acting at a point, the angle between the first and second being 30°, that between second and third 50°, that between third and fourth 75°.

2. If 15 lbs. are represented by a line $2\frac{1}{2}$ in. long, what will a line 6 in. long represent? How long a line will be required to represent 108 lbs.?

3. From a straight rod, *AB*, 6 ft. long, forces act up and down. The forces up are as follows: 6 lbs. at distance 1 ft. from *A*; 8 lbs. $1\frac{1}{2}$ ft. from *A*; 12 lbs. 1 ft. from *B*. The forces down are as follows: 18 lbs. 3 ft. from *A*; 5 lbs. 2 ft. from *A*; 10 lbs. at *A*. Represent this system by diagram drawn to scale, stating scales used. (Use separate scales for *force* scale and *apparatus* scale.)

4. Draw to scale lines representing force of 2000 lbs. pulling a car straight along a track, a force of 300 lbs.

holding it back, and a force of 80 lbs. pushing it sidewise against the rails.

5. (*a*) Show by diagram to scale an inclined plane 50 ft. long, rising 30° from the horizontal.

(*b*) Draw height and base of this plane. Determine by measurement the length of each.

(*c*) Show a force of 36 lbs. acting at angle of 20° with the face of the plane.

16. Free Body.—As stated in the preceding article, forces are the actions exerted by bodies upon one another. We can apply forces to a body only by getting other bodies to act upon it.

It is often convenient in mechanics, however, to disregard the sources of the forces acting and to conceive of a body from which all external bodies that act upon it have been removed and their places taken *by the force* which they exerted. In such cases it is convenient to include all reactions, whether positive or passive, among the forces, giving them their proper direction with respect to the body under consideration, and assigning to them a value equal to their opposite force action. A body thus regarded *as acted upon by forces only*, is called a "*free body.*"

Thus in the case of the lever *AC* used in applying forces to a screw jack, Fig. 12, we may, when convenient, regard all supports, etc., as removed and their places taken by the "*forces*" with which they act upon *AC*.

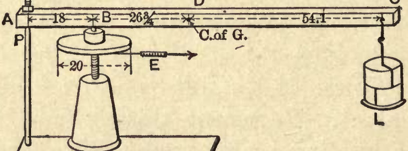


Fig. 12.

We then have the "free body" and arrangement of forces shown in Fig. 13. The elastic reaction of the head is

replaced by the force R, and this with the applied load L, the pull of the rod at A and the weight W of the lever, are all the forces acting upon the lever. They are, of course, merely shown *qualitatively* here. If the load L, dimensions of the lever, etc., be known, as in cases to be considered later, all forces may be determined in amount, and the lines for P, R, W, and L may be drawn to scale, showing the amounts of the forces.

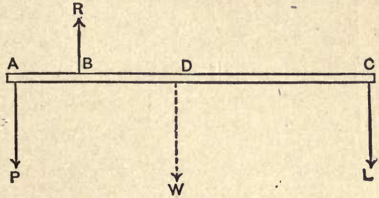


FIG. 13.

The pulley at L in the derrick, Fig. 15, is shown as a "free body," in Fig. 14.

In larger structures containing several parts, any particular member may be taken as the "free body" as desired.

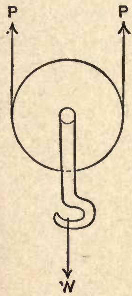


FIG. 14.

Thus in the derrick of Fig. 15, for example, we may consider the jib CD as a "free body," the other parts being replaced by their forces, as in Fig. 16 (a). T indicates the force applied by the pull in the tie DE, L the force due to the load at D, R the reaction at C, W the weight of the jib, and P the tension in the hoisting rope running from D to the drum of the hoisting engine. (This rope is not shown in the photograph.) If we neglect friction, P is evidently equal to $\frac{1}{2}L$.

With a known load at L, known angles, etc., the amounts of these forces may be found as we shall see later.

In many cases the *exact direction* of the reaction R may not be known. It is evident, however, that the hinge at C keeps the end of the jib from dropping down or being pushed

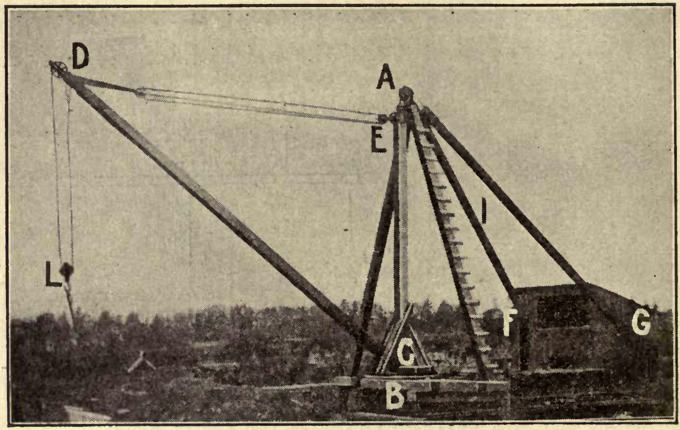


Fig. 15.—Derrick Car.

toward the right. These two effects of the pin may, in such cases, be shown in place of R, as V and H of Fig. 16 (b).

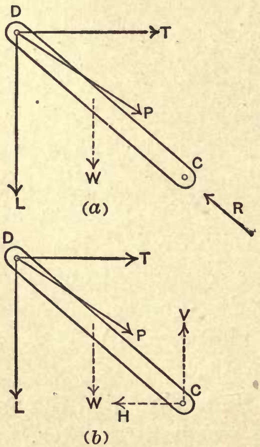


Fig. 16.

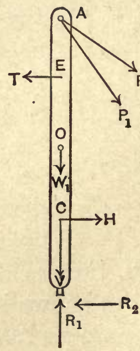


Fig. 17.

It is evident from the preceding that the jib is under compression.

If we regard mast AB as the free body, we have the arrangement of forces shown in Fig. 17. T, H, and V are the forces exerted *upon the mast* by the tie DE and jib CD respectively. They are the *equal and opposite reactions* to the forces exerted by the mast on the jib, as in Fig. 16. Note their opposite directions in the two figures.

R_1 and R_2 are the floor reactions upon the mast, P_1 and P_2 the pulls exerted by the guys AF and AG, and W the weight of the mast.

The tie ED regarded as a free body has the forces C, L, and T, acting upon it as shown in Fig. 18. The effect of C and L is that of a single force T_1 and ED is therefore a simple tension member.

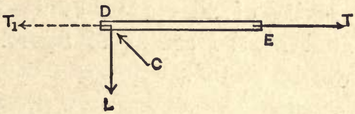


FIG. 18.

If desired, the *pin* at D alone may be taken as the free body, when the force diagram of Fig. 19 results.

Or, if more convenient, the crane may be considered as a whole to be one body and the force diagram of Fig. 20 constructed. The *external* forces are R_1, R_2, L, P, P_1, P_2, and weights of parts W and W_1. The *internal* actions at the pin C and in the tie ED need not be considered, for each consists of an equal, opposite, pair which, therefore, just annul one another, as far as external effect upon the whole crane is considered.

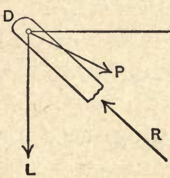


FIG. 19.

The student should study these diagrams of the forces for the members of a structure, very carefully, and should note particularly the dependence of the direction of the force upon the particular member considered. He should construct diagrams for various given structures, until

thoroughly familiar with the graphical representation of force conditions at any selected point. Upon the thorough comprehension of these ideas depends the success with which the solution of subsequent problems in mechanics may be attempted. Where the weight of any part is included among the forces acting upon the body it is to be considered as a single force acting at a point called the *center of gravity* of the body. The center of gravity of all regular bodies is situated at the center of figure. In other cases its location will be stated.

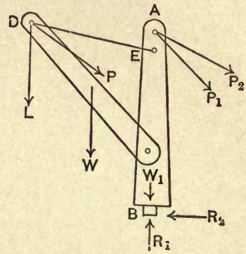


Fig. 20.

EXERCISES

Qualitative Force Diagrams for a "Free Body"

(Point of application and direction of forces to be shown, but lines need not be drawn to scale to show value.)

1. Construct force diagrams for the problems 1–13 of Chapter I.

2. Construct a force diagram for a body W being drawn with uniform speed upon an incline rising 40° to the horizontal.

3. Construct force diagrams for the following, as "free bodies:"

(*a*) Arm AB of wall crane, Fig. 81.

(*b*) Boom BA and tie BC of model of hoisting crane, Fig. 76.

(*c*) Stick CB and tie AB of the apparatus shown in Fig. 72.

(*d*) Mast EO of crane, Fig. 85.

(*e*) Boom of derrick, Fig. 88.

CHAPTER III

COMPOSITION AND RESOLUTION OF FORCES, VELOCITIES, ETC.

17. Composition of Velocities.—In a preceding paragraph it was pointed out that a body might have one velocity with reference to a second body and the second body might have some other velocity with reference to a third. In such cases it is often important to determine what the velocity of the *first* body is with reference to the third. This velocity can be determined by properly combining the first two velocities. The velocity obtained in this way is called the *Resultant Velocity,* and the process by which it is obtained is called the *Composition of Velocities.* The two velocities from which the resultant is obtained are called the *Component Velocities.* The resultant velocity shows the direction and rate of motion of a body to which two velocities different in amount or direction or both are simultaneously imparted.

Two cases arise in the composition of velocities:

CASE I.—*The velocities are along the same straight line.* It is evident here, from a little consideration, that if the two component velocities are in the *same direction* their resultant equals their sum. If the two velocities are in *opposite* directions their resultant equals their difference. Or, to make a general statement which applies to the composition of *any number of velocities in the same straight line, whether in the same direction or opposed: The resul-*

tant is always equal to the algebraic sum of the component velocities.

Thus, if a man walk at the rate of 4 miles per hour down the aisle of a car going 20 miles per hour, his velocity with respect to the ground will be: (1) If he walks in the direction the car is going, $4+20=24$ miles per hour; (2) if he walks in the opposite direction, $20-4=16$ miles per hour. Or, calling motion in direction of car *positive*, motion in contrary direction *negative*, we have for the first case $(+4)+(+20)=+24$, *the sign showing the direction of the resultant* velocity; for the second case $(-4)+(+20)=+16$. A *minus sign* before the resultant would indicate that the combined result gives a motion in opposite direction to that of the car. Thus in (2), if the velocity of the car were only 2 miles per hour, we should have as our expression $(-4)+(+2)=-2$, or the man is traveling 2 miles per hour with respect to the ground, and in a direction opposite to that in which the car is going.

Case II.—*The velocities are along lines making an angle with each other.* The component velocities must here be added *geometrically instead of algebraically.* This process will be explained in the following discussion of the parallelogram of velocities.

18. Parallelogram of Velocities.—In Fig. 21 imagine the point O to be moving with a velocity Y along the blade of the T-square AB. If the T-square is at rest, in one unit of time the point will move along the path OP to P. If, on the contrary, point O is at rest on the blade, but the whole T-square slides along the drawing board to a new position CD, with a velocity X, in one unit of time O will pass along the path OR to R. Finally, if point O moves along the blade with a velocity Y *and at the same time* the T-square slides along the drawing board with the velocity X, in one unit of time the point O will be

carried along the board a distance OR and across the board a distance OP, or it will be at Q, which is the farther extremity of the diagonal of the parallelogram $OPQR$, shown dotted in Fig. 21.

If a unit of time half as big had been chosen, point O would have moved half as far along the blade, and the T-square would have carried it half as far along the board. The point O, therefore, in half the time would have moved to S. If the unit of time is still further subdivided, other points, as T, W, etc., may be located. *All these points will lie on the diagonal OQ of the parallelogram.* Hence the path traveled by the point O to which the two velocities X and Y are imparted, is along the diagonal of the parallelogram; and since the position Q is reached in unit time, the length of this diagonal, OQ, represents correctly the resultant velocity of the point.

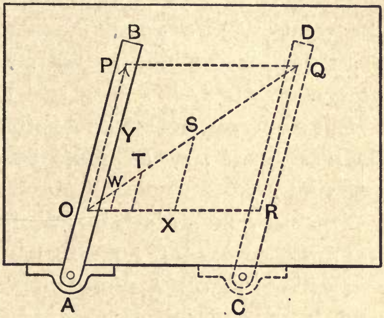


FIG. 21.

Therefore, to find the resultant velocity of a body to which two component velocities not in the same straight line are imparted, represent the two component velocities in their proper directions and to a convenient scale, as the adjacent sides of a parallelogram. Complete the parallelogram, and from the point of intersection of the sides representing the component velocities, draw the diagonal. This diagonal represents the resultant velocity both in direction and in amount, in accordance with the scale assumed.

19. Composition of Forces.—The principles just stated for velocities apply to the composition of any quantities

which may be represented by lines, i.e., which have *amount* and *direction.* The case of chief importance in mechanics is the *composition of forces.*

When two or more forces act at a single point * on a body, there is a single force which will produce the same effect as the several forces. This single force is called the *resultant force.* Any one of the original forces is called a *component force.*

If the component forces all act in the same straight line, the *resultant force equals their algebraic sum.* Its direction is readily determined as for velocities by a proper observance of signs.

If two forces act at one point but in different directions, their resultant may be found by application of the parallelogram law.

This may be stated, in full, for forces thus:

Parallelogram of forces: IF TWO FORCES ACTING AT A SINGLE POINT ON A BODY BE REPRESENTED IN MAGNITUDE AND IN DIRECTION BY TWO STRAIGHT LINES DRAWN FROM THAT POINT, AND IF A PARALLELOGRAM BE CONSTRUCTED ON THESE LINES AS ADJACENT SIDES, THE *resultant* OF THE TWO FORCES WILL BE REPRESENTED IN BOTH MAGNITUDE AND DIRECTION BY THE *diagonal* OF THIS PARALLELOGRAM, DRAWN FROM THE POINT.

This proposition may be regarded as a corollary of the parallelogram of velocities, since we know that forces may be measured by the velocities which they will impart in a given time.

In Fig. 22 let AB and AC represent two forces P_1 and P_2, acting at the point A on some body; then according to *some* scale AB will also represent the velocity which the force P_1 would impart in one unit of time to the body

* NOTE.—The point of application of a force may be considered as being *at any point along the line of action of the force.*

if acting alone; and AC will also represent the velocity which the force P_2 would impart to the body in the same unit of time if it were to act alone. The resultant of these two velocities, AB and AC, is the diagonal of the parallelogram AR, that is, the two forces acting together will impart to the body in one unit of time a velocity equal to AR. But a force equal to AR would also impart this same velocity. Therefore, the force AR, the diagonal of the parallelogram, produces the same effect as the two original forces, P_1 and P_2, and consequently is their resultant.

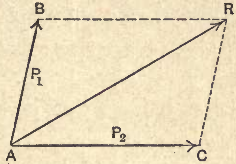


FIG. 22.

20. Composition of More than Two Forces.—Any number of forces *acting at a single point and lying in the same plane,* may be combined by repeated applications of the parallelogram law. In this method, the resultant of two of the forces is first found, then the resultant of this first resultant and the next force is found, and so on, until, all the forces have been combined.

NOTE.—*Computation of the Resultant of Two Forces.* (*a*) *Forces acting at* 90°. If the parallelogram constructed in the composition of two forces is also a rectangle, half of the parallelogram forms a right triangle, as triangle ABC, Fig. 23. Since side $BC = AD =$ force F_2, it is obvious that $R^2 = F^2_1 + F^2_2$.

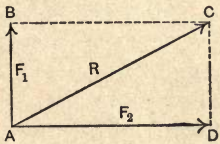


FIG. 23.

(*b*) Forces acting at *any* angle. Forces a and b acting at the acute angle x are given; required to *compute* their resultant R.

Prolong side AD and drop a perpendicular, p, from C to this line. Let distance from D to foot of perpendicular at $E = q$. Then ACE is a right triangle, and $R^2 = (b+q)^2 + p^2$. Therefore,

$$R^2 = b^2 + 2bq + q^2 + p^2.$$

But $q = a \cos x$ (see Appendix) and $p^2 = a^2 - q^2$. Substituting those values in preceding equation,

$$R^2 = b^2 + 2ab \cos x + q^2 + a^2 - q^2,$$

or,

$$R^2 = a^2 + b^2 + 2ab \cos x.$$

Hence, the square of the resultant of two forces equals the sum of the squares of the two forces plus twice their product into the cosine of the angle included between their action lines.

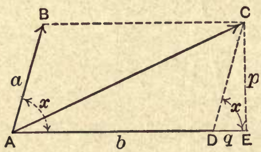


Fig. 24.

This equation, *written here for an acute angle*, is perfectly general, as will be seen from the following cases:

(1) a and b act in the same straight line and in same direction. The included angle, x, is now zero, and since $\cos 0 = 1$, our equation becomes

$$R^2 = a^2 + b^2 + 2ab,$$

or,

$$R = a + b.$$

(2) a and b act in a straight line, but in opposite directions. x now $= 180°$, and since $\cos 180° = -1$, we have

$$R^2 = a^2 + b^2 - 2ab,$$

or,

$$R = a - b.$$

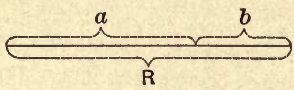


Fig. 25.

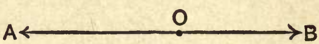


Fig. 26.

(3) a and b act at 90°. Cos x now equals zero, and

$$R^2 = a^2 + b^2$$

as for a right triangle.

(4) a and b act at an obtuse angle. Cos x is now negative with a value somewhere between 0 and -1. Therefore our equation becomes

$$R^2 = a^2 + b^2 - 2ab \cos x.$$

PROBLEMS IN COMPOSITION OF VELOCITIES AND FORCES

SOLVE GRAPHICALLY

1. A man is rowing with a velocity of 6 miles an hour at right angles to a current which has a velocity of $2\frac{1}{2}$ miles an hour. Find his velocity with reference to the shore.

2. Find graphically the resultant of the following pairs of forces:

(*a*) $P_1 = 36$ lbs. and $P_2 = 24$ lbs.; angle between $=$ 30°.
(*b*) $P_1 = 45$ lbs. and $P_2 = 18$ lbs.; angle between $=$ 45°.
(*c*) $P_1 = 22$ lbs. and $P_2 = 50$ lbs.; angle between $=$ 60°.
(*d*) $P_1 = 32$ lbs. and $P_2 = 14$ lbs.; angle between $=$ 120°.
(*e*) $P_1 = 25$ lbs. and $P_2 = 48$ lbs.; angle between $=$ 75°.

3. Draw an angle of 40° and upon its sides lay off forces of 8 and 6 units respectively. Find their resultant.

4. Forces of 15 and 6 grams act from a point at an angle of 60°. Find the resultant and the angle between resultant and each force.

5. Resultant is a force of 25 lbs. Force a equals 12 lbs. Angle between a and resultant is 30°. Find force b and angle between a and b.

6. Two forces act at an angle of 60°. Their resultant is 40 lbs. and one of the forces is 25 lbs. Find the other force.

7. Three posts are placed in the ground so as to form an equilateral triangle. An elastic cord is stretched around them, the tension of which is 25 lbs. Find the force pressing on each post and its direction.

8. The water is pressing horizontally against the side of a dam with a force of 225 tons. The weight of the dam is 850 tons. Find the direction and the magnitude of the resultant.

9. A building is 220 ft. high, with an exposed side 110 ft. long. In a storm the wind pressure on the side of this

building is 50 lbs. per sq.ft. The weight of the building is 10,000 tons. Find the direction and the magnitude of the resultant.

10. Suppose a man is walking at the rate of 3 miles an hour on the deck of a steamer in a direction at right angles to its length, and suppose the steamer is moving through the water with a velocity of 12 miles per hour in an easterly direction, and suppose there is a current which is running at the rate of 4 miles per hour in a southwesterly direction. Find the velocity of the man with reference to the shore.

11. A train is moving at the rate of 35 miles an hour in a northerly direction. The wind is blowing at the rate of 22 miles per hour from the southwest. Find graphically the direction in which a weather vane would point if it were attached to the top of the train.

12. In Example 11, suppose the wind is blowing with a velocity of 40 miles per hour from the southwest. How fast must the train move in order that the weather vane shall point towards the northwest?

13. Suppose a man is rowing at the rate of 4 miles an hour and wishes to cross a stream at right angles to the bank; suppose also that there is a current of $2\frac{1}{4}$ miles per hour. In what direction must he row with reference to the current in order that he may accomplish his purpose?

14. A boat is moored in a stream by two ropes, one fastened to either bank. The ropes make an angle of 90° with each other. The force of the stream on the boat is 450 lbs., and the pull on one of the ropes is 200 lbs. Find the pull on the other.

SOLVE BY COMPUTATION

15. Two forces, 60 and 91 lbs., act at an angle of 90°. Compute their resultant.

16. Two forces act at an angle of 90°. Their resultant is 200 grams and one force is 85 grams. What is the other force?

17. Find the resultant of forces of 20 and 16 lbs. acting:

(*a*) At an angle of 30° with each other.
(*b*) At an angle of 70° with each other.
(*c*) At an angle of 120° with each other.

21. The Resolution of Forces.—Any quantity having both *amount and direction*, as a given motion, velocity, or force, may be resolved into *two or more components* which will produce the same effects as the given quantity. This process is known as the resolution of forces (or motions, etc.).

Thus, in Fig. 27, if OR is a given force, the component forces represented by the adjacent sides of any of the parallelograms which may be formed on OR as a diagonal, such as OA and OB, or OX and OY, will have OR for their resultant and will therefore be in every way equivalent to OR, and may be at any time substituted for it.

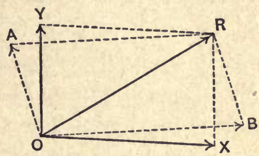


Fig. 27.

As an indefinite number of parallelograms may be drawn with OR as the diagonal, there will be an indefinite number of sets of components into which every force may be resolved. It is evident, therefore, that in order to resolve a force into definite components, enough data must be given to determine the form of the parallelogram. Thus if components a and b (See Fig. 28), are to make *known angles* with R, as, say, 30° and 40°, draw R to proper scale, then from O draw lines of indefinite length, in the proper directions for a and b. Then from extremity D of R, draw DN and DM parallel respectively to a and b. These will cut off lengths from O, determining values of components a and b.

If R is to be resolved into forces a and b, and *either* a

or b is known both in amount and direction, points D and either M or N are fixed; from which parallelograms may be completed and a and b both determined.

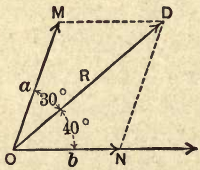


FIG. 28.

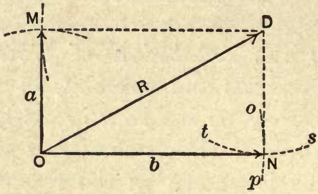


FIG. 29.

If components a and b *are known in amount but not in direction,* first draw R to scale. Then from O, with radius b, describe arc op (see Fig. 29), and from D with radius equal to a, describe arc st. Their intersection fixes point N. Locate point M in a similar way, and complete the parallelogram. Then measure the angles between a and b, or of either with R.

22. Resolution into Rectangular Components.—The most important case of the resolution of forces, velocities, etc., is that in which the given quantity is resolved into two components which are at right angles with one another. Such components are called *rectangular components.* For example, we frequently desire to know the *horizontal component* and the *vertical component* of a given force.

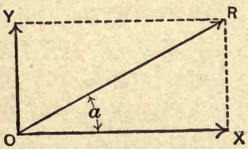


FIG. 30.

Suppose OR, Fig. 30, to be the force. Construct the parallelogram $OXRY$ so that OY will be vertical and OX will be horizontal; then the horizontal component of OR will be the vertical projection of OR, or OX, and the vertical component of OR will be the horizontal projection of OR, or OY.

It will be seen from the figure that component OX gives *all the effect* of OR in the horizontal direction—*no more and no less*—and that component OY gives the *total vertical effect.*

23. Use of Squared Paper in Resolving a Force into Rectangular Components.—Where a given force is to be resolved into rectangular components graphically, the most convenient method is to draw the force parallelo-

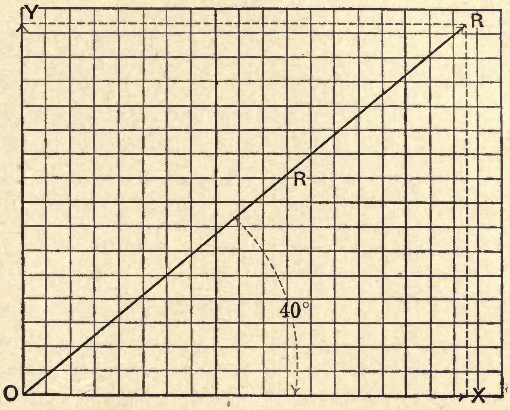


FIG. 31.

gram on squared paper; the components may then be read directly from the ruled paper. Thus, in Fig. 31, R is a force of 120 lbs. acting at an angle of 40° to the horizontal; to find its vertical and horizontal components.

Suppose the paper is ruled in $\frac{1}{8}$ in. squares: a convenient scale is then $\frac{1}{8}$ in. = 5 lbs. Drawing a line OR, 40° to the horizontal, and 3 in. long, to represent the force, and reading from the paper the value of the components, we have

Horizontal component $X = 18.5$ divisions $= 92.5$ lbs.

Vertical component $Y = 15.4$ divisions $= 77.0$ lbs.

This method makes it unnecessary to actually measure OX and OY, to determine their length, or to use a protractor, etc., in order to get the components at right angles. Nor is it necessary to draw the dotted lines showing the components; these lines are drawn for Figs. 31, 32, and 33 merely to make the explanation clearer.

24. Composition of Several Forces by the Method of Resolution along Rectangular Axes. — As suggested in Art. 20, the resultant of several forces acting at a point and in the same plane, may be found by first determining the resultant of any two, then the resultant of this resultant and a third force, etc., until all the forces have been used. A simpler method, however, is suggested by the preceding article. Thus, suppose four forces act at a point O. Force F_1 of 32 lbs. acts at an angle of 20° with the horizontal; force F_2 of 20 lbs. acts at an angle of 40° with F_1; force F_3 of 18 lbs. acts at an angle of 60° with F_2, and force F_4 of 40 lbs. acts at an angle of 110° with F_3. Required to find the amount and direction of their resultant. Draw the forces in the proper direction upon squared paper, ruled to, say, $\frac{1}{16}$ inch, using any convenient scale, as, for example, $\frac{1}{16}$ inch = 2 lbs., and determine the horizontal and vertical components of each force (Fig. 32).

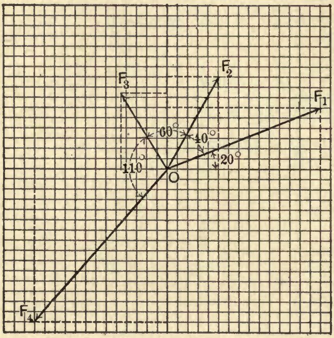


FIG. 32.

Since the components are equivalent to the original forces and can replace them in every way, forces F_1, F_2, F_3, and F_4 can be replaced by *eight* forces, viz., two horizontal

Force	Horizontal Comp.			Vertical Comp.		
	Direction.	Length.	Value.	Direction.	Length.	Value.
F_1	Right	15.0 div.	30.0 lbs.	Up	5.5 div.	11.0 lbs.
F_2	Right	5.0 div.	10.0 lbs.	Up	8.6 div.	17.2 lbs.
F_3	Left	4.5 div.	9.0 lbs.	Up	7.8 div.	15.6 lbs.
F_4	Left	12.8 div.	25.6 lbs.	Down	15.2 div.	30.4 lbs.

forces to the right from 0 of 30 lbs. and 10 lbs. respectively, and two horizontal forces to the left of 9 lbs. and 25.6 lbs. respectively, three vertical forces upward of 11 lbs., 17.2 lbs., and 15.6 lbs. respectively, and one vertical force downward of 30.4 lbs. We may now combine the horizontal forces directly to find their resultant, which is

$$+30 \text{ lbs.} + 10 \text{ lbs.} - 9 \text{ lbs.} - 25.6 \text{ lbs.} = +5.4 \text{ lbs.},$$

or, 5.4 lbs. to *the right* from *O*.

Combining the vertical forces similarly, we have

$$+11 \text{ lbs.} + 17.2 \text{ lbs.} + 15.6 \text{ lbs.} - 30.4 \text{ lbs.} = +13.4 \text{ lbs.}$$

or, 13.4 lbs. *vertically upward* from *O*.

The original four forces are therefore equivalent to two forces, one of 5.4 *lbs. horizontally to the right from O, and one of* 13.4 *lbs. vertically upward from O.*

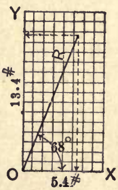


FIG. 33.

Constructing a force parallelogram with these two forces as components (Fig. 33), and using a larger scale for convenience ($\frac{1}{16}$ in. = 1 lb.), we find R, the resultant of these forces and therefore also of the *four original forces,* F_1, F_2, F_3, and F_4, to be very nearly 15 lbs., acting at an angle which, measured with a protractor, is found to be approximately 68° with the horizontal.

PROBLEMS

Resolution of Forces, Motions, Etc.

SOLVE GRAPHICALLY

1. A force of 75 lbs. has two components, a and b, making angles of 30° and 45° respectively with the force. Find values of components.

2. Resolve a force of 100 lbs. into two components, one of which shall be a force of 60 lbs. acting at an angle of 25° with the 100 lbs. force.

3. Resolve a force of 35 lbs. into two components of 20 lbs. and 25 lbs., and determine the angle between the 35 lbs. force and each component.

4. A body weighing 16 lbs. rests upon a smooth surface inclined 30° from horizontal. Find the pressure perpendicular to the surface and the force parallel to the surface.

5. A canal boat is towed by a pull of 120 lbs. at an angle of 10° with axis of the boat. Find, using squared paper:

   (a) Force urging boat directly ahead in line of its axis, and
   (b) Force urging boat in toward shore.

6. The rod R is thrust against AC (Fig. 34) by a force of 10 lbs. straight along R. What must be the force in direction AC to prevent slipping?

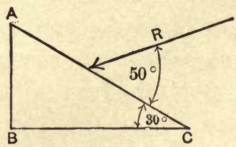
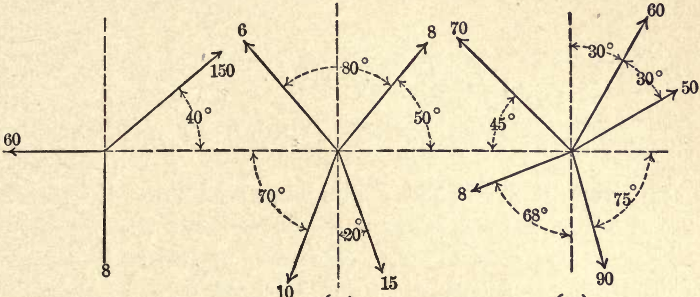


Fig. 34.

7. Find the amount and direction of the resultant of the combinations of forces, shown in Fig. 35, using the method of resolution into X and Y components. Use

squared paper as directed in Arts. 22 and 23, and tabulate data neatly.



(a) (b) (c)

FIG. 35.

CHAPTER IV

EQUILIBRIUM. THREE FORCES AT A POINT

25. Definition.—When a body is acted upon by forces, the effect of which is the *same as if no force acted so far as change in* the existing *state of motion* of the body is concerned, the forces are said to be *balanced,* or *in equilibrium,* and THE BODY IS SAID TO BE IN EQUILIBRIUM. The following illustrations will make clearer the meaning of this statement:

1. Suppose a heavy weight, W, rests upon a horizontal plank, AB, Fig. 36. The forces acting upon W are a vertical gravity pull, G, and the reaction, R, due to the elastic properties of the plank. As W is supported, $R=G$; and therefore since the forces are oppositely directed their resultant (algebraic sum) *is zero.*

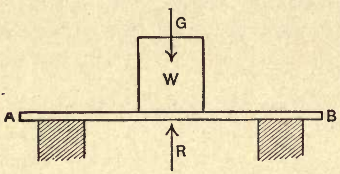


FIG. 36.

There is thus no unbalanced force acting on W, and *so far as motion is concerned,* its condition is the same as if no forces were applied to it. *W is therefore in equilibrium.**

2. Or, suppose a *horizontal pull, P,* is moving W *at uniform speed* along AB, Fig. 37. The forces in the ver-

* NOTE.—The *changes of form* produced in W and in plank AB are not here considered. The bodies are assumed *to be rigid.* In Chapter XII the effects of force in producing *change of form* will be studied.

tical plane are again a gravity pull, G, and the reaction, R, and, as before,

$$R-G=0.$$

The horizontal forces are P and the reaction due to friction, or Fr. Since P is just able to keep W moving uniformly, $P=Fr$ or $P-Fr=0$. The resultant of all the horizontal forces on W is therefore zero, and the resultant of all the vertical forces is zero; and as there is no horizontal or vertical component, there can be *no resultant force anywhere in the plane.* All the forces on W are therefore *balanced*, and thus have *no effect upon W as far as change of motion is concerned.* Hence, if W were at rest it would remain at rest, and if once set moving, it will continue to move in the same straight line without change in velocity. *W is therefore in equilibrium.*

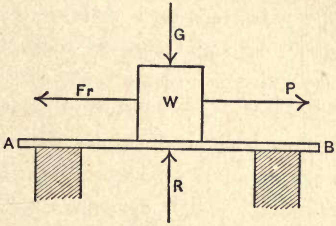


FIG. 37.

If pull P exceeded friction, W would move in the direction of P with constantly increasing velocity; if P were less than Fr, W would move slower and slower and finally stop. In either case its *state of motion would be changing and W would not be in equilibrium.*

3. Example (2) might be extended to the case of a *body rotating at a uniform speed,* as, for example, a shaft driven by a belt. When the driving effect of the belt and the retarding effects of friction, etc., are equal, the shaft is in equilibrium, and will continue to rotate at the same speed. When the driving effect exceeds the retarding, the shaft speeds up; when the reverse is true, it slows down.

Thus, A BODY AT REST, OR IN UNIFORM MOTION IN A STRAIGHT LINE OR ROTATING AT UNIFORM SPEED, IS IN EQUILIBRIUM.

26. Application to Engineering Structures.—Engineering structures, as the various types of trusses, bridges, cranes, etc., are designed to support, rigidly, the loads applied to them. It is evident, therefore, that the component parts of such structures must be held at rest or, as we may now say, *in equilibrium*—in other words, that the loads applied at any point or points must set up such tensions and compressions throughout the structure that the forces applied to each particular member *shall be balanced,* thus holding that member in equilibrium. If we wish, therefore, to determine the stresses in the members of such structures, we have only: First, *to represent a selected member or part as a "free body"* (see Chapter II, Section 16); and then, second, *to determine what values the forces applied to such part must have in order that, acting in the fixed directions, they may balance each other.* Thus in the derrick of Fig. 15, we may find the tension in the tie ED and the compression in the jib CD, by determining the values of the force upon the jib to hold it in equilibrium, i.e., values of T and R, Fig. 16, for a given value of L in order that *the forces may be balanced.* The solution of all such problems, therefore, depends upon the application of the fundamental laws for the equilibrium of particular combinations of forces. These laws will be discussed and their applications to practical structures will be pointed out in the succeeding chapters.

27. Equilibrium with Two Forces Acting.—It will be evident from our definitions and discussion of equilibrium that a *body acted upon by a single force, or by a system of unbalanced forces* (i.e., a system whose resultant is *not zero*), *can not be in equilibrium.*

The least number of forces which may produce equilibrium is therefore two. It is obvious also that *a body acted upon by two forces can be in equilibrium only when the forces are equal in amount and act in opposite directions in the same straight line.*

Thus the suspended body shown in Fig. 38, if acted on by *two forces only,* (i.e., no friction at pin P), could be in equilibrium only when the pull of gravity G is in the same vertical line with the reaction R of the pin. In any other position G would have a moment about P which would cause rotation with increasing speed.

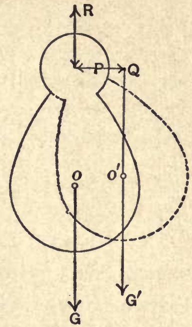


Fig. 38.

The column, shown in Fig. 3, and the "tie," ED, of the derrick, Fig. 15, are illustrations of typical "two-force pieces." Such members are always either simple tension or compression members, with the action line of the forces along the axis of the piece. The student should note, here, that when we are considering *the forces which two-force members exert on the other parts of the same structure,* such forces are always *both toward* or *both away from* the joints at the ends of the member. Thus in Fig. 3, the column is *pushing upward* at A and *pushing downward* upon the support at B. The tie in Fig. 15 is *pulling* on the mast AB *in the direction* ED and *also pulling* on the jib DC in the *direction* DE.

28. Three Forces in a Plane, Acting at the Same Point. —Forces whose action lines pass through a common point are called *concurrent forces.*

Let a and b, Fig. 39, be two forces whose action lines intersect at O. Combining these by the parallelogram law, we find their resultant R. If a third force is to act

with a and b in such a way that the three shall balance, or in other words, so that the combined effects of a, b, and c *shall be zero*, it is evident that c *must be equal and opposite to resultant* R and that it must act in the same straight line as R. Force (c), equal and opposite to the resultant R of forces (a) and (b), is sometimes called the *equilibrant* of forces (a) and (b). Hence three forces acting at a point and in the same plane will produce equilibrium when

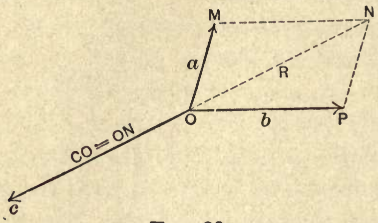


FIG. 39.

THE DIAGONAL OF THE FORCE PARALLELOGRAM FORMED ON ANY TWO FORCES AS SIDES, DRAWN FROM THE POINT OF CONCURRENCE, IS EQUAL AND OPPOSITE TO THE THIRD FORCE.

NOTE.—*Experimental Test of Parallelogram Law.*—The truth of the parallelogram law may be easily tested experimentally as shown in Fig. 40. A ring O is supported by two cords OA and OB, in which spring balances, P_1 and P_2 are inserted; and from the ring is suspended a weight, W. Hold a drawing board covered with paper behind the apparatus and indicate center of ring O by a pencil dot. Place dots also back

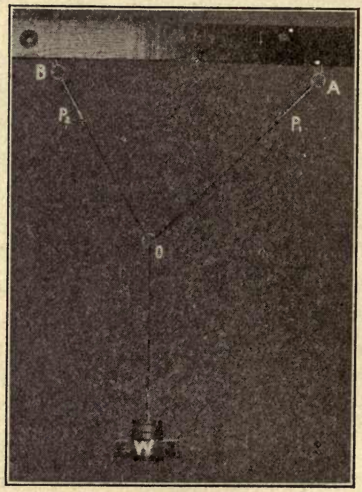


FIG. 40.—Apparatus for test of parallelogram law.

of the centers of cords OA and OB and OW. Connect these dots by lines showing the directions of the forces, and record beside each line the value of the force in that direction as determined from the balances P_1 and P_2 and W. The parallelogram on lines OA and OB may now readily be drawn to scale. Its diagonal expressed in force units should equal W, and its direction be in the vertical line OW. Similarly, the diagonal of the force parallelogram on OA and W as sides, should be equal and opposite to the force P_2, and the diagonal of the force parallelogram on OB and W should be equal and opposite to P_1.

This condition may be used for the *graphical solution* of any problem in which three forces act at one point, provided data for the construction of the parallelogram is obtainable.

Example 1.—Fig. 41 shows a laboratory model of a simple truss construction in which the horizontal member AB hinged or pinned at A, and bearing a load W, is supported by a tie CB. Suppose $W=70$ lbs. Required: (*a*) Tension in the tie BC; (*b*) thrust of AB against the wall. It is obvious that the point B is here in equilibrium under the action of three concurrent forces, viz.: 1st. Vertical downward pull of 70 lbs.; 2d. Pull in the tie which must be in the direction BC (for if tie were cut it is evident that B would move downward); and 3d. The reaction of stick AB which is along AB and evidently in the direction indicated by R (for if AB be cut, point B will fall inward toward the wall. Reaction R must be equal and opposite to resultant of forces W (70 lbs.), and tension in BC.

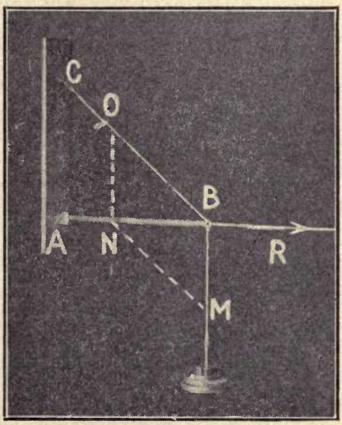


FIG. 41.—Simple Truss. Laboratory Method.

We are thus to construct a parallelogram whose sides shall have the directions BW and BC and whose diagonal shall have the direction BA, the *side* BW *being known in amount.*

Lay off a distance BM which shall represent 70 lbs., by some convenient scale, e.g., 1 in. = 10 lbs. From M draw a line parallel to the tie to intersect the action line of the force along the stick, and from this point, N, draw NO parallel to BM to intersect the action line of force in BC. BO represents tension in BC to same scale that $BM =$ 70 lbs. Measuring this, suppose we have $BO = 9\frac{3}{4}$ in. approximately, or tension in BC is 97.5 lbs.

Reaction $R =$ resultant BN. Suppose BN measures $6\frac{1}{2}$ in. approximately, then reaction $R = 65$ lbs. And since reaction R and thrust against pin at A are equal (Why?), thrust at $A = 65$ lbs.

Example 2.—In the simple " A " truss, a model of which is shown in Fig. 2, members AB and CB are under compression and are supporting the load at B. Three forces act on the pin: the thrusts of members AB and CB, and the load L. The directions of the forces, and suggestions for the force parallelogram are given in the diagram for Fig. 4. The student should be able to complete the construction and determine the compression in AB and CB. The thrusts of these members at A and C may then be resolved into their horizontal and vertical components and the results may be checked with the balance readings at D and D'. In structures of this kind, in which the weights of the members must be allowed for, half the weights of AB and CB are to be added to the load W at B, the other halves regarded as acting at A and C. The latter should be added to the vertical components of the thrusts before comparing with the balance reading D'. The horizontal base of this truss is provided as

a guide to keep the plane of the truss vertical. One end of the truss (C) is held by a pin, the other end (A) is held "free" by the balances D and D'. If the truss is symmetrical, the vertical component at C will equal the corresponding component at A. If not symmetrical, after checking for A, end A may be pinned and end C then freed and the vertical component measured.

Example 3.—The forces acting upon a body supported upon an inclined plane are shown in the model of Fig. 42.

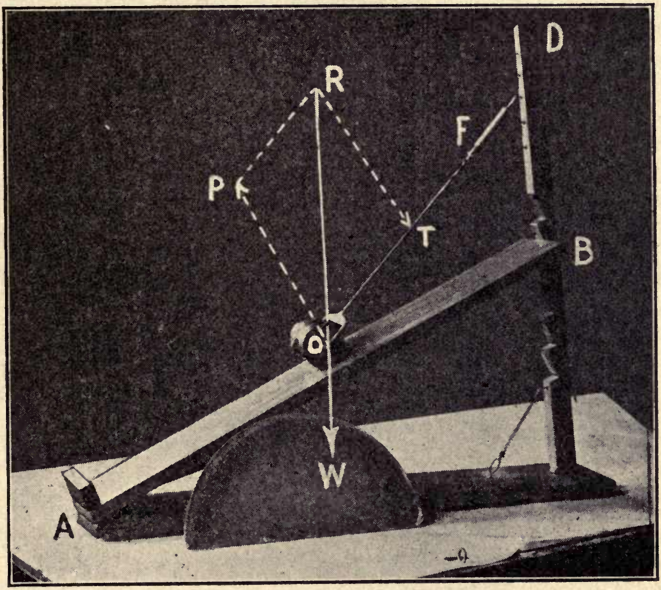


FIG. 42.—Inclined Plane and Roller.

The construction is here such that friction of the roller on the plane is negligible, hence the reaction P of the plane is perpendicular to the surface. Three forces, the pull F, P, and the weight W act upon the roller, holding

it in equilibrium. The directions of the three forces are all known for any given setting of the plane. Since W must be equal and opposite to the resultant of forces F and P, if a force OR be drawn to scale to represent W in amount, the force parallelogram may be completed, having OT and OP as adjacent sides. These lines may then be measured and the values of F and P determined. F may then be checked with the balance reading in the cord, and P by pulling with a balance in the direction OP until the roller is just free from the plane. This apparatus represents the common case of a body supported on, or being drawn at uniform speed up an incline.

PROBLEMS

Three Forces Acting at a Point

SOLVE GRAPHICALLY

1. Forces of 20 and 36 lbs. act at an angle of 60°. What third force acting with these will produce equilibrium?

2. A rigid, weightless rod AB, hinged to a wall at end A, is held inclined 40° to the vertical by a cord running horizontally from end B to a point on the wall vertically above the hinge. A weight of 10 lbs. is hung at end B. Find tension in the cord and thrust of stick against the wall at the hinge.

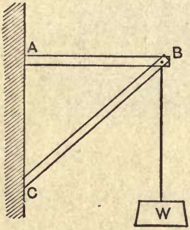


Fig. 43.

3. If the breaking strength of the cord in problem 2 is 30 lbs., what weight hung at B will be just large enough to break the cord?

4. In the apparatus of Fig. 43.

$$AB = 6 \text{ ft.}$$
$$AC = 8 \text{ ft.}$$
$$\angle CAB = 90°.$$

If $W = 14$ lbs., find the tension in AB and thrust in CB.

5. If in the apparatus shown in Fig. 72, angle $CBW=10°$, angle $BAC=50°$, and weight $W=400$ lbs., find stresses in BC and BA, and the vertical and horizontal reactions of the pier at C and at A.

6. In Fig. 44 $W=150$ lbs. Find thrusts in BA and BC and the reactions horizontally and vertically at A and C.

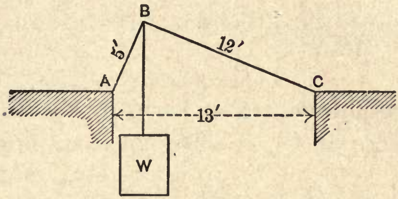


Fig. 44.

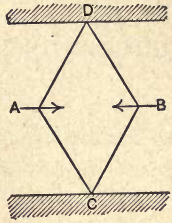


Fig. 45.

7. $ABCD$, Fig. 45, is a frame of four sides having equal lengths and pivots at the four corners. If angle ACB is 60°, find the thrust in each arm when A and B are drawn together by a force of 10 lbs., and find also the force separating C and D.

8. In the truss, Fig. 2, AB and BC make angles of 30° with the horizontal. The weight L is 640 lbs. Find the stress in AB and BC. Also find the tension in the tie AC.

9. A tight-rope walker weighing 150 lbs. stands in the middle of a tight-rope 20 ft. long. The rope is depressed 1 ft. Find tension in the rope.

10. The anchor rope of a balloon makes an angle of 70° with the ground. If the "lifting force" of the balloon is 300 lbs., find the tension in the anchor rope and the horizontal force exerted against the balloon by the wind.

CHAPTER V

EQUILIBRIUM: PARALLEL FORCES. CENTER OF GRAVITY

29. Conditions of Equilibrium for Parallel Forces.—Suppose the rod AB, Fig. 46, to be acted upon by forces a, b, c, etc., which are parallel. It is evident that the effect of these forces may be any of the following: (1) To displace AB either upward or downward; (2) to cause AB to rotate about some point; or, (3) to produce both displacement and rotation at the same time.

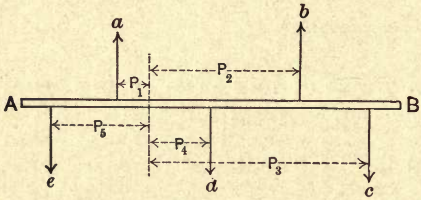


FIG. 46.

Therefore, in order that AB may be in equilibrium, the tendencies toward displacement and rotation must each be so BALANCED AS TO HAVE A TOTAL RESULTANT OF ZERO.

For convenience, forces acting upward may be considered as +forces, forces downward as −forces; moment of force tending to produce rotation *counter-clockwise as a +moment,* that tending to produce rotation *clockwise as a −moment.* Then there will be no displacement of AB either upward or downward when *forces upward=forces downward*; i.e., $a+b=c+d+e$; or, when the total effect of forces is zero, i.e., $a+b-c-d-e=0$.

We may also determine the perpendicular distance from each force to ANY POINT *along* AB, as P, and thus find the moments of the forces with respect to point P. It is evident that forces c, d, and a tend to produce rotation clockwise about P, forces b and e tend to produce motion counter-clockwise. When these tendencies are equal, i.e., when

$$b \times P_2 + e \times P_5 - a \times P_1 - c \times P_3 - d \times P_4 = 0,$$

there will be no rotation.

The conditions of equilibrium for parallel forces are therefore:

1. SUM OF ALL FORCES $= 0$.
2. SUM OF THE MOMENTS OF ALL FORCES $= 0$.

30. Three Parallel Forces.—Equilibrium with three parallel forces acting is a very common and important case in mechanics. Suppose a and b, Fig. 47, to be two parallel forces acting in the same direction on a weightless rod, AB; it is required to find a third force, c, which, acting with a and b, will produce equilibrium. By condition (1),

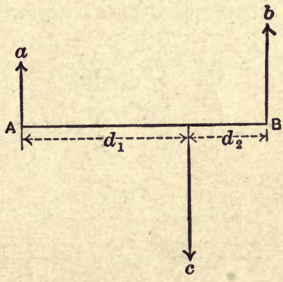


FIG. 47.

$$a + b - c = 0.$$

Since a and b act in same direction, c must act in the opposite direction and must equal $a + b$. By condition (2), taking moments about P the point of application of c,

$$b \times d_2 - c \times 0 - a \times d_1 = 0.$$

Therefore,

$$b \times d_2 = a \times d_1,$$

or, force c must act at such a point as to *divide the distance between a and b into parts d_1 and d_2, which are inversely proportional to the magnitudes of the forces.*

If the forces act in opposite directions, as a and c, Fig. 47, the third force b which will produce equilibrium equals $c-a$, and must be applied *on the opposite side of c* (the greater force) at a distance such that $\frac{\text{force } a}{\text{force } b}=\frac{d_2}{d_1}$.

31. Resultant of Parallel Forces.—Since force c (Fig. 47) produces equilibrium with forces a and b, *it must be equal and opposite to the combined effects of a and b, i.e., to their resultant.*

Therefore the following statements follow from the preceding:

First: *The resultant of two parallel forces acting in the same direction* acts in a direction parallel to them both and *is equal to their sum.*

Second: *This resultant acts at such a point as to divide the perpendicular distance between them in two parts which are inversely proportional to the two forces.*

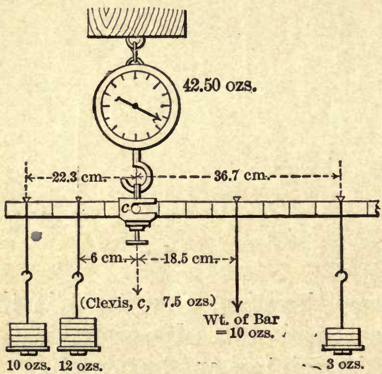


FIG. 48.

The student should formulate similar statements for the case where a and b act in opposite directions.

32. Apparatus for Studying the Laws for Parallel Forces.—A convenient laboratory apparatus for testing the conditions stated for the equilibrium of parallel forces is shown in Fig. 48. A meter rod bearing movable steel knife-edges is suspended from brass clevis C. The upward

force is measured by a 5-lb. spring balance graduated to $\frac{1}{2}$ oz. Downward forces are applied by scale pans and weights suspended from small steel knife-edges, which bear upon the bar at definite points. The weights of the meter bar, bearings at C, and knife-edges must be included in the downward forces. One, two, or more upward forces may be used as desired.

Tests should be made for algebraic sum of the forces and for the sum of the moments about, (1) one end of the bar, (2) point of application of an upward force, (3) any point along the bar where no force is applied.

33. Solution of Problems.—The following problems will illustrate the application of the conditions of equilibrium for parallel forces.

Example 1.—Find the pressure on each of the two supports of the beam shown in Fig. 49. The *force diagram* is drawn below the *apparatus* diagram.

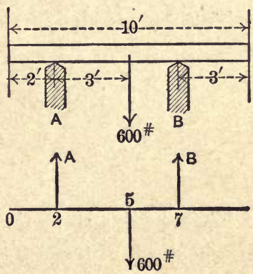


FIG. 49.

The equation of forces is,

$$A+B=600 \text{ lbs.},$$

or

$$A+B-600=0.$$

For the equation of moments it is best to take moments about the point of application of one of the support reactions, since in that case the moment of that reaction is zero, and the unknown value of that reaction does not appear in the moment equation. Taking moments about point of application of A, we have

$$-(3\times 600)+5\,B=0$$

Hence, $B=360$ lbs. Then from the first equation,

$$A+360-600=0;$$

$$A=240 \text{ lbs.}$$

Example 2.—Find amount, direction, and point of application of resultant of the system of parallel forces shown in Fig. 50. Since this resultant is equal and opposite to the force that will produce equilibrium (Art. 31), we may most conveniently apply the laws for equilibrium of parallel forces and find this force E which may be called the equilibrant. A force *equal to this force,* applied at the same point, *but in the opposite direction,* is the required resultant. The equation of forces gives

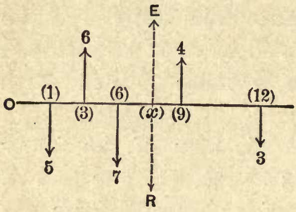


FIG. 50.

$$6+4+E-5-7-3=0.$$

Therefore, the equilibrant is an upward force of 5. And hence the *resultant* is a downward force of 5.

Taking moments about 0, we have, if x = distance of E from 0:

$$-(5\times1)+(6\times3)-(7\times6)+5x+(4\times9)-(3\times12)=0,$$

$$5x=29,$$

$$x=5.8.$$

The resultant is thus found to be a force 5 acting down at a point 5.8 from 0.

34. Couples.—*Two equal and parallel forces acting in opposite directions are called a* COUPLE. (See Fig. 51.) The perpendicular distance a between the two forces is called the *arm* of the couple.

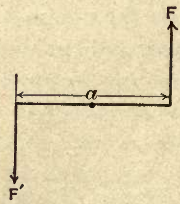


FIG. 51.

Since $F=F'$ it is evident from the laws which apply to parallel forces that:

1. *A couple can produce rotation only.* The moment of

a couple is equal to the *product of the arm by one of the forces.*

2. *A couple cannot be balanced by a single force.* In order to produce equilibrium a second couple having an equal moment is required.

35. Center of Gravity.—*The center of gravity of a body is the point at which we may assume the force of gravity to always act*: that is, it is the *point of application of the resultant of all those little parallel forces which gravity exerts on the particles of the body regardless of the way in which the body may be placed or turned.*

If a body is homogeneous and symmetrical in shape, the position of its center of gravity will be evident from inspection. Thus the center of gravity of a uniform timber is on its central axis at the middle of its length, the center of gravity of a rectangle at the intersection of its diagonals, of a circle at the intersections of two diameters, etc.

36. Center of Gravity of a Triangle.—The center of gravity of the triangle ABC, Fig. 52, from symmetry will lie along the line BD, bisecting side AC, but it will also lie on the line AE which bisects the side BC. The point of intersection of BD and AE is therefore the center of gravity of the triangle. This point will be found to be one-third of the distance from the middle point of any side of the triangle to the apex opposite it.

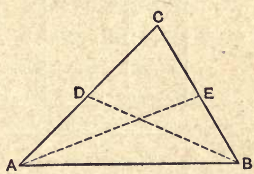


FIG. 52.

37. Center of Gravity of any Figure.—Given a card of the shape and dimensions of Fig. 53; required its center of gravity.

Assume an axis XX' (here for convenience one edge of card) and let C be the center of gravity at a distance Y_0 from this axis.

If the card is of uniform thickness and material, the weight of each of the three rectangles of which it is composed will be proportional to their areas, and the weight of the whole proportional to its total area. Since a force equal to its weight acting at the center of gravity of the card will support it, we may write as our equation of moments about the XX' axis,

$$40 \times Y_0 - (24 \times 1.5) - (4 \times 5) - (12 \times 8) = 0;$$

$$Y_0 = 3.8 \text{ inches.}$$

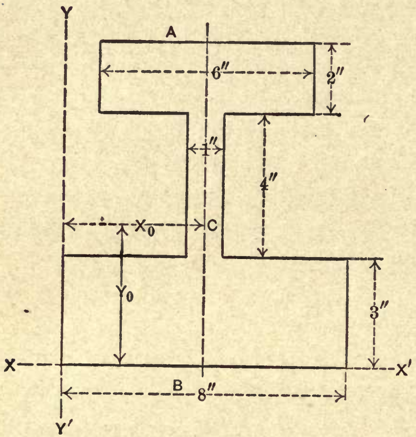


Fig. 53.

Or, the center of gravity of the card is somewhere on a line parallel to axis XX' and 3.8 inches from it.

From the symmetry of the card, we know that the center of gravity will be located on center line AB. Hence center of gravity of the card is at a point C on AB, 3.8 inches from B.

If the figure were not symmetrical, we could take moments about an axis YY' at right angles to XX', and determine the distance X_0 from this axis to the center of gravity C.

Thus for Fig. 53,

$$40X_0 - (24 \times 4) - (4 \times 4) - (12 \times 4) = 0;$$

$$X_0 = 4 \text{ inches from } YY'.$$

Or, center of gravity C is at a point 3.8 inches from XX' and 4 inches from YY'.

From the preceding it is evident that the center of gravity of any figure may be found from the general equations:

$$(1) \quad AX_0 = \text{sum } (ax),$$

$$(2) \quad AY_0 = \text{sum } (ay).$$

Where A is the total area of the figure, X_0 the distance from center of gravity of the figure to an assumed axis YY', Y_0 the distance from center of gravity to an axis XX' at right angles to YY'' and sum (ax) and sum (ay) the sum of the products of each portion of the figure multiplied by the distance of its center of gravity from the Y and X axes respectively.

The student should test this general method by computing the center of gravity of various thin laminas cut from cardboard and then comparing the computed result with the experimental result found, as in the succeeding article.

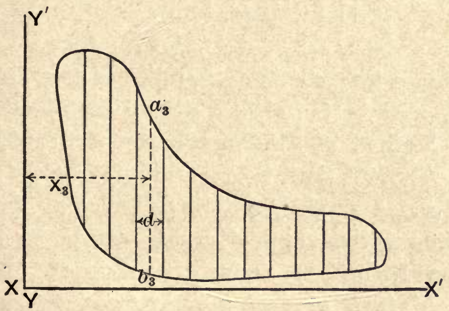


Fig. 54

An interesting case is furnished by bodies bounded by curved lines, as Fig. 54. The area of the figure or its parts cannot be computed readily in this case. A result sufficiently good for practical use may, however, be obtained by dividing the figure into narrow elements of equal width by lines drawn parallel to the YY' axis. Then if the elements are suffi-

ciently narrow, the length of the central line of each, as a_3b_3 in figure, will be very nearly the *average length* of the element, and this times the width will give the area. Then, if d be the width of each element,

$$\begin{aligned}\text{Sum } (d\times a_1b_1+d\times a_2b_2+d\times a_3b_3+\ldots)\\ =\text{sum of areas of all the elements}\\ =\text{area of whole figure.}\end{aligned}$$

If x_1, x_2, etc., be distances from YY' axis to central line of each element,

Sum $(d\times a_1b_1\times x_1+d\times a_2b_2\times x_2+\ldots)=$ sum of moments of all the elements about the YY' axis.

Then, by the general equation above,

Area whole figure $\times x_0=$ sum of moments of all elements, or,

$$x_0=\frac{\text{Sum } (d\times a_1b_1\times x_1+d\times a_2b_2\times x_2+\ldots)}{\text{Sum } (d\times a_1b_1+d\times a_2b_2+\ldots)}.$$

By dividing the figure into elements parallel to the XX' axis, the value of Y_0 may be found in a similar manner.

38. Experimental Method of Finding Center of Gravity.—It was shown in Art. 27 that the center of gravity of a body, suspended from a single point in such a way that it can turn freely about the support, must always be at some point in the vertical line under the support. By suspending a body in turn at two points, two verticals containing the center of gravity may be determined, their intersection therefore gives the center of gravity of the body.

PROBLEMS

1. Find force X for the system shown in Fig. 55.

2. A bar AB, 14 ft. long, is pivoted at B. A weight of 18 lbs. is hung from A. Find force which must be applied 5 ft. from A to produce equilibrium.

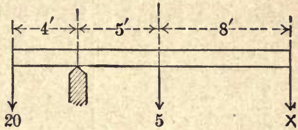


FIG. 55.

3. Two men carry a weight of 180 lbs. between them on a pole 5 ft. long. Where should the weight be hung in order that one may carry three times as much of the weight as the other?

4. A rod AB, the weight of which may be neglected, is supported $2\frac{1}{2}$ ft. from end A. If the rod is 10 ft. long and 100 lbs. are hung from B, what force will be required at A for equilibrium? What will be pressure on the support?

5. Parallel forces of 75 lbs. and 30 lbs. act in the same direction, 20 ft. apart. Find amount and point of application of their resultant.

6. A rod 6 ft. long, the weight of which may be neglected, rests upon two supports placed under the ends. Where must a weight of 22 lbs. be hung in order that pressure on one support may be 9 lbs.?

7. You have a rod 2 ft. long and of negligible weight and a balance whose maximum capacity is 64 ozs. Show two ways in which you could arrange your apparatus in order to weigh with this balance a body weighing 14 lbs. Give distances on your diagrams and proof of the correctness of your methods.

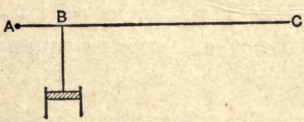


FIG. 56.

8. Safety valve lever AC, Fig. 56, 2 ft. long, and weighing 12 lbs., is pivoted at A. Its center of gravity is 10 in. from A. Weight of valve V is 8 lbs., diameter of valve 3 in.

Valve is pivoted at B, 4 in. from A. Find pressure per square inch required to open the valve, when a weight W of 150 lbs. is suspended from end C of lever.

9. A straight, uniform lever AB, 12 ft. long, balances about a point 5 ft. from B, when weights of 9 and 13 lbs. are suspended from A and B, respectively. Find weight of lever.

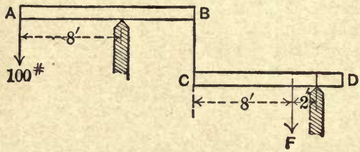


FIG. 57.

10. AB, Fig. 57, is lever 14 ft. long, and weighing 12 lbs. CD is a lever 12 ft. long and weighing 8 lbs. Assuming weights of levers to act at their middle points, what force can be exerted at F (8 ft. from C) by a force of 100 lbs. applied at A?

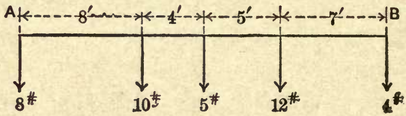


FIG. 58.

11. Find point of application of resultant of the forces in Fig. 58.

12. A uniform beam AB, 20 ft. long, weighing 600 lbs., is supported by props placed under its ends. Four feet from prop A a weight of 200 lbs. is suspended, and 6 ft. from B a weight of 500 lbs. Find pressure on each prop.

13. A rod whose weight is 10 lbs. and length 4 ft. is supported horizontally by a smooth peg at one end and a vertical string 15 in. from the other end. Calculate the tension in the string.

14. Find the center of gravity of a board, 8 ft. long, 8 in. wide at one end, 4 in. wide at the other, tapering equally on each side.

15. Compute center of gravity of an iron bolt having following dimensions: Head $\frac{7}{8}'' \times \frac{7}{8}''$ and $\frac{7}{16}''$ thick; threaded part $\frac{7}{16}''$ diameter and $2''$ long.

16. Compute the pressure on the head of the jack, Fig. 12, when 150 lbs. are hung at C, if the bar weighs 40 lbs.

17. Compute center of gravity of the areas shown in Figs. 59–62.

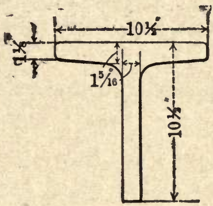


FIG. 59.

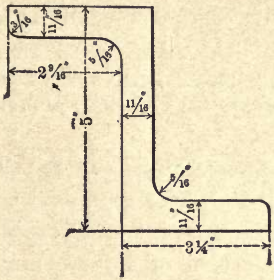


FIG. 60.

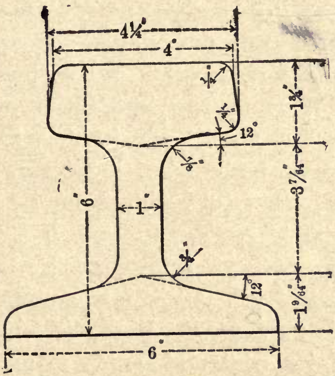


FIG. 61.

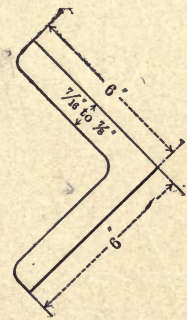


FIG. 62.

CHAPTER VI

CONCURRENT FORCES APPLIED AT SEPARATE POINTS. ALGEBRAIC CONDITIONS FOR EQUILIBRIUM

39. Three Forces not Parallel, Applied at Different Points of a Body.—The graphical solution for three forces acting at the same point is given in Chapter IV. Since the point of application of a force may be regarded as *at any point in the line of action of the force*, the same solution is also applicable to cases of bodies acted upon by three forces not parallel and applied at different points on the body.

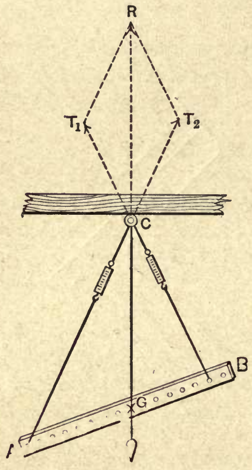


FIG. 63.

Thus in the illustration shown in Fig. 63, in which AB is any heavy body (e.g., a picture, bar, or other object), supported by flexible cords, AC and BC, which are not parallel, three forces act upon AB producing equilibrium, viz.: the weight W of AB acting at G, its center of gravity, and the pulls applied by AC and BC.

The forces in AC and BC lie along the cords and therefore their action lines, if continued, must intersect at some point C in the figure. These forces are, therefore, equivalent to a single force (their resultant) acting through C,

and hence if the third force acting on AB (its weight W), is to produce equilibrium with the forces in AC and BC, it *must also pass through* C and be equal and opposite to the resultant of the first two. Or, in general, no matter where on the body the forces are applied,

If a body acted upon by three forces in one plane and not parallel be in equilibrium:

(*a*) THE ACTION LINES OF THE FORCES MUST ALL PASS THROUGH A COMMON POINT. And

(*b*) THE DIAGONAL OF THE PARALLELOGRAM FORMED ON ANY TWO FORCES AS SIDES AND DRAWN FROM THE POINT OF CONCURRENCE MUST BE EQUAL AND OPPOSITE TO THE THIRD FORCE.

In the case of a suspended body, as Fig. 63, therefore, no matter what the lengths of the supporting cords AC and BC may be, the center of gravity of the body *must lie in a vertical line through* C, the point of intersection of the cords continued. To find the tensions T_1 and T_2, in the cords for a given weight of AB, and given angles, we may thus assume the common point C, as the point of application of the forces of our force parallelogram.

Then to complete the construction, draw the vertical line CR to represent weight W, according to a convenient scale, and construct the parallelogram having CR as a diagonal and its adjacent sides in the directions CT_1 and CT_2. The values of forces T_1 and T_2 are then determined by the lengths of the lines forming the sides of this parallelogram.

If cords AC and BD are divergent, point C will lie below the bar. The construction for tensions in the cords is similar to Fig. 63, however, and should be made without difficulty by the student.

As a further illustration, suppose a ladder or timber AB, of known weight, leans against the vertical side of a smooth

building at any given angle θ to the side, as in Fig. 64. Required to find the pressure against the building and the amount and direction of the ground reaction at A. Neglecting the friction at B against the building, the reaction H of the building will be horizontal (i.e., there will be no vertical force (friction) to prevent sliding). Three forces therefore act upon the ladder, viz.: the reaction H of the house, the weight W of the ladder, and a force G equal to the ground reaction. H and W intersect at O, hence ground reaction G *must* pass through this point for equilibrium (i.e., to be equal and opposite to resultant of W and H). In other words, the ground both holds vertically up on the ladder (V), and keeps the end from slipping through friction (Fr).

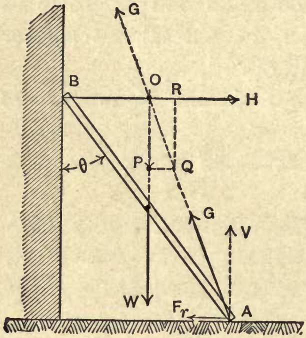


FIG. 64.

We now know the *directions* of OP and OR of our force parallelogram, the direction of the diagonal OQ, and also the amount of force OP $(=W)$. Therefore, laying off OP of a length to represent W, we may complete the parallelogram to scale and determine force $OR=$ reaction of house and force $OQ=$ ground reaction. G is, of course, in the opposite direction to OQ.

A very common example of equilibrium with three forces is furnished by the bent arm lever used in operating switches, semaphores, etc., on railroads. A pull F_1, Fig. 65,

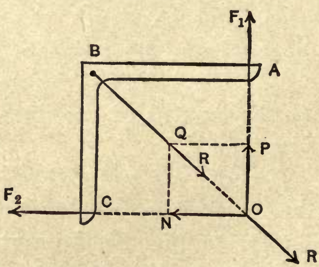


FIG. 65.

is applied to rotate the lever; this gives rise to force F_2, which operates the signal. At the same time a pressure is set up against the pin at B about which the lever rotates. At any given position, the lever is held in equilibrium (neglecting friction) by three forces, F_1, F_2, and the reaction R of the pin. Any one being known the others may be determined. The construction of the force parallelogram will be clear from the figure.

Fig. 66 shows a form of student laboratory apparatus designed to illustrate the principle of the bent arm lever. The brass lever with arms A and B is made large enough at portion D to turn upon ball bearings resting upon the

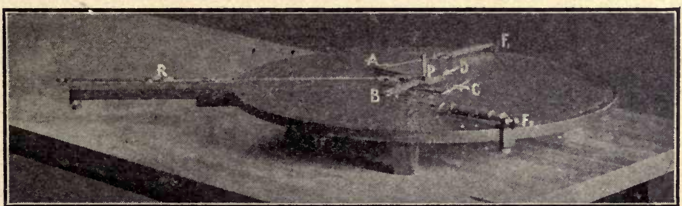


FIG. 66.—Bent Arm Lever.

brass plate C. D is weighted until the lever will rest horizontally upon the ball bearings. The whole is mounted upon a circular board. A pin is inserted through the lever at P, and held firmly in the plate to furnish the axis about which the lever may turn.

A *known* force is applied at F_1 by adjusting the clamp F_1. It is then required to find:

(*a*) The force at F_2.

(*b*) The *amount* and *direction* of the pressure on pin P.

The clamp F_2 is adjusted, and the swinging arm R rotated about the board until the *amount and direction of*

force at R *is such that the lever remains at rest when pin* P *is withdrawn.*

Neglecting friction, which is here very small because of the ball bearings, three forces in a plane are acting upon the lever producing equilibrium. (Weight of lever is here borne by reaction of the support and hence *this pair of balanced* forces need not be considered.) The student is expected to complete the construction, and

1. Prove that the unknown pin reaction always lies along a line from the point of concurrence of the action lines of the forces at F_1 and F_2.

2. Determine required forces at F_2 and R by means of a force parallelogram. These may then be checked by comparison with the balance readings.

40. Triangle of Forces.—Referring to Fig. 39, it will be seen that the three forces, a, b, and c, are equal in magnitude and parallel in direction to the three sides of the triangle OMN. It follows, therefore, that *if three forces in the same plane and acting at one point are in equilibrium they may be represented in magnitude and in direction by the three sides of a triangle taken in order.* This principle is known as the *triangle of forces.* It is merely another form of statement of the parallelogram of forces which is convenient as a test for equilibrium.

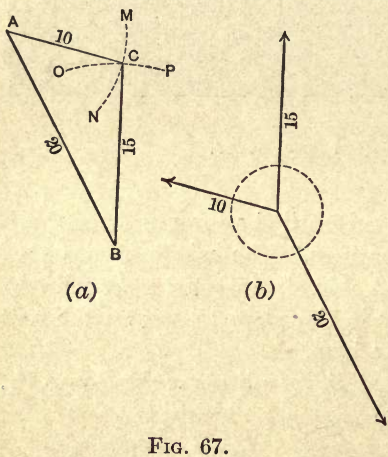


FIG. 67.

Example 1.—In the simple truss, Fig. 41, for example, the three forces acting are parallel to the sides of the

apparatus triangle ABC. The forces bear the same relation to each other therefore as the sides of triangle ABC. Or, W: tension in BC: thrust in $BA = CA:CB:BA$.

Example 2.—It is required to find the angles at which forces of 10, 15, and 20 lbs. must act to produce equilibrium. First lay off a line AB to represent a force of 20 lbs. From A with a radius to represent 10 lbs. describe arc MN, and from B with radius to represent 15 lbs. describe arc OP to intersect this at C. Then AB, BC, and CA (Fig. 67*a*), represent the directions of the forces, and Fig. 67*b* shows the angles between them.

41. Polygon of Forces.—An extension of the triangle of forces gives the principle of the polygon of forces. Thus, suppose F_1, F_2, F_3, F_4, F_5 be five forces acting at a point P, Fig. 68. By the triangle of forces, forces F_1, F_5, and r_1 will produce equilibrium; or r_1 is equal and opposite to the resultant of F_1 and F_5. Similarly, r_2 will produce equilibrium with F_2 and r_1, and hence with F_1, F_2, and F_5; and, if F_4 forms a *closed triangle* with forces r_2 and F_3, F_4 will produce equilibrium with r_2 and F_3 and therefore with F_1, F_2, F_3, and F_5, hence the five forces will be in equilibrium. Therefore:

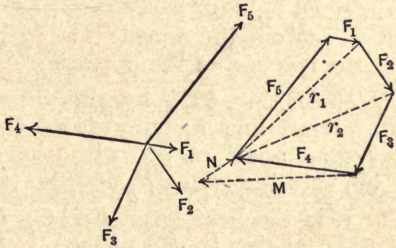


FIG. 68.

If any number of forces acting at a point and in the same plane are in equilibrium they may be represented by the sides of a closed polygon taken in order.

If force F_4 *does not form a closed triangle* with r_2, and F_3 (in other words, if the five forces do not form a closed polygon), but lies in the position of M in the figure, F_4

will not produce equilibrium with r_2 and F_3, and the *five forces will not be in equilibrium.* A sixth force, N, will be required for equilibrium.

PROBLEMS

(Solve Graphically)

1. Will forces of 5, 6, and 14 lbs. produce equilibrium?

2. Find the angle at which the concurrent forces 10, 12, and 7 must act to produce equilibrium.

3. Forces of 8, 14, and 10 lbs. act from a point and produce equilibrium. Find the angle between the 8-lb. force and the 10-lb. force.

4. What horizontal force at the axle will be required to draw a wheel of radius 2 ft. and weight 20 lbs. over an obstacle 6 in. high? (When free from the ground, three forces act on the wheel: Its weight, the horizontal pull, and the reaction of the obstacle.)

5. If AB, Fig. 63, is a uniform metal bar weighing 50 lbs., and plumb line hanging from C, the point of support, makes an angle of 30° with the cord CA, and an angle of 40° with the cord CB, find the tensions in CA and CB.

6. If in the bent lever, Fig. 65, BC is 12 in. long, and is at right angles to BA, which is 8 in. long, and if force $F_1=8$ lbs. is parallel to BC and force F_2 is parallel to BA, find force F_2 and the direction and magnitude of the reaction at pin B.

7. Stick AB, Fig. 64, is 20 ft. long and makes an angle of 70° with the ground at A. Its weight is 100 lbs. Find reactions at house and at ground.

42. The Use of Simple Trigonometric Functions.—It is frequently necessary in practical mechanics to determine the horizontal and vertical components of a given force, to compute the moment arm of a force with respect to a

given axis, to determine various dimensions of a structure from other specified dimensions and angles, etc. These quantities are so readily found through the use of simple trigonometric functions that although the student may have no previous knowledge of trigonometry, the use of such functions is here introduced. The necessary ideas are so few, so simple, and so readily acquired, that time is saved in the end. For definition of sine, cosine, and tangent of an angle, see Appendix.

The diagonal of any rectangle, as BC, Fig. 69, divides the rectangle into two right triangles. By definition,

$$\frac{\text{side } AC}{\text{side } BC} = \text{sin of angle } ABC.$$

Therefore, $$AC = BC \text{ sin angle } ABC.$$

And $$BC = \frac{AC}{\text{sin angle } ABC}.$$

If the length BC is known and also the angle ABC, the length of AC may be computed by multiplying the length BC by the value of the sine of the angle ABC, taken from the table of sines. Or, if AC is known, BC may be computed by dividing AC by the sine of angle ABC. Similar equations may be written from the definitions of the cosine and tangent of the angle. All these equations are equally true for *forces* AC, BC, etc., for if *ABC is a force triangle* we know from the parallelogram law that the forces are correctly represented in amount by the lengths of the sides. Further applications of the trigonometric method are shown in the following examples

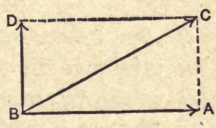


FIG. 69.

Examples.—(See Fig. 69).

(*a*) Given $BC=15$ and angle $ABC=40°$. Find length of AC.

$$AC=BC \sin 40°$$
$$=15\times.643$$
$$=9.64.$$

(See table of functions, Appendix, p. 316.)

(*b*) Given $BA=25$ and angle $ABC=60°$. Find BC.

$$\frac{BA}{BC}=\cos 60°,$$
$$\frac{25}{BC}=.500,$$
$$BC=50.$$

(*c*) Given $AC=9$ and $BA=13$. Find the angle ABC.

$$\tan ABC=\frac{AC}{BA}$$
$$=\frac{9}{13}=.692.$$

By reference to the table of tangents we find that the angle whose tangent is .692 is between 34° and 35°.

(*d*) Referring to Fig. 30, suppose force OR to be 60 lbs. and angle a, between its line of action and the horizontal, to be 36°. It is required to find the horizontal and vertical components of OR.

Horizontal component $OX=OR\cos 36=60\times.809=48.5$ lbs.

Vertical component $OY=XR=OR\sin 36=60\times.588$
$=35.3$ lbs.

EXERCISES

(Solve by Use of Trigonometry)

1. A force of 50 lbs. acts in a direction inclined 20° to the horizontal. Find values of its X and Y components.

2. A pole 20 ft. long leans against the vertical side of a house. If the pole makes an angle of 24° with side of house:

(*a*) How far from the foot of the pole to the house?

(*b*) How far up the side of the house does the pole touch?

3. A horse pulls with a force of 700 lbs. on traces making an angle of 10° with the horizontal. What is the force pulling the load horizontally, and what is the lifting force upon the load?

4. A ladder leans against a house with its foot 8 ft. from the house. The angle between the ladder and the ground is 75°. How long is the ladder?

5. In a pillar crane, such as shown in Fig. 74, jib AB is 16 ft. long. What is the height of the mast if, when angle CAB is 40°, tie BC is horizontal? What is length of tie in this position?

6. A safe weighing 9000 lbs. is being rolled up planks which are inclined 36° with the horizontal. What is the perpendicular pressure of the safe on the planks? What is the force with which it tends to roll down the planks?

43. Composition of Forces by Use of Trigonometry.— Instead of the graphical determination of the horizontal and vertical components of a system of forces by aid of squared paper, as in Art. 23, we may compute such components by aid of the functions of the given angles. Thus, referring to Fig. 32,

Horizontal component $F_1 = 32 \cos 20° = 32 \times .940 = 30.1$ lbs.

Vertical component $F_1 = 32 \sin 20 = 32 \times .342 = 10.9$ lbs., etc.

These components may then be combined just as before for Fig. 32. The table of functions given in the Appendix is correct to the nearest unit in the third place, therefore computed components may be assumed to be correct to three significant figures. This is a more rapid, and also a more correct method, than is usually possible where graphical methods are used for determining the values of the forces.

44. General Conditions of Equilibrium for Concurrent Forces.—In Art. 23 it was shown that, IF ANY NUMBER OF FORCES ACT AT A COMMON POINT, AND LIE IN THE SAME PLANE, WE MAY REPLACE ALL THESE FORCES BY TWO FORCES LYING AT RIGHT ANGLES TO ONE ANOTHER, WHICH ARE RESPECTIVETY EQUAL TO THE SUM OF ALL THE HORIZONTAL COMPONENTS OF THE ORIGINAL FORCES, AND THE SUM OF ALL THE VERTICAL COMPONENTS OF THE ORIGINAL FORCES. In other words, we may replace the original forces by

> Sum X and sum Y, where sum X = algebraic sum of all horizontal components, sum Y = algebraic sum of all vertical components.

It is evident from Fig. 33, that if sum $X = 0$ and sum $Y = 0$, *OR, the resultant of the whole system of forces, must be zero.* The forces would then have no effect upon the motion of the body upon which they act, and the body would be in equilibrium. Hence,

A BODY ACTED UPON BY ANY NUMBER OF FORCES ACTING AT ONE POINT AND IN THE SAME PLANE WILL BE IN EQUI-

LIBRIUM WHEN, IF THE FORCES BE RESOLVED INTO THEIR X AND Y COMPONENTS,

$$(1) \qquad \text{Sum } X=0.$$

AND

$$(2) \qquad \text{Sum } Y=0.$$

45. Analytical Method of Solution for Forces in Simple Structures.—The conditions of equilibrium just stated furnish the simplest method for the determination of stresses in the members of simple jointed structures. Such simple structures are made up of tension and compression members, riveted or pinned together at the joints. The forces lie along the axis of the members, and in cases where the weight of the parts is included, a half weight of each member entering a joint is to be added to the load at that point. As examples of such solutions the following typical cases are given in detail:

Example 1.—A body weighing 100 lbs. is being moved at a uniform speed along a rough horizontal surface by a force of 40 lbs. acting at an angle of 30° with the surface. Required the force of friction and the vertical pressure on the table.

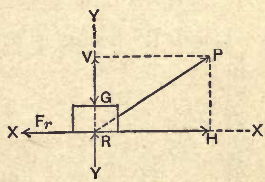


FIG. 70.

Consider the body as a "free body," i.e., table removed and its place taken by R its vertical reaction, and Fr the friction. (See Fig. 70.) Resolve force P into its horizontal and vertical components H and V. Then, by the conditions of equilibrium,

$$(1) \qquad H-Fr=0. \quad (\text{Sum } X=0.)$$

$$(2) \qquad R+V-G=0. \quad (\text{Sum } Y=0.)$$

Substituting the known value for G, and the values for

H and V obtained by resolving the force P, the unknown forces Fr and P may be found.

Thus, $H=40 \cos 30°$, and $V=40 \sin 30°$, and we may write the equivalent equations:

$$(3) \qquad 40 \cos 30° - Fr = 0.$$

From which, $\qquad Fr = 34.6$ lbs. Friction.

$$(4) \qquad R + 40 \sin 30° - 100 = 0.$$

From which,

$R = 100 - 20 = 80$ lbs., vertical reaction of the table.

Example 2.—In the simple truss, Fig. 71, tension T in tie CB may be resolved into its vertical and horizontal components V and H, where $V = T \sin \theta$ and $H = T \cos \theta$. Then, from the conditions of equilibrium for the pin at B regarded as a free body, we may write as our equations:

(1) $\qquad$ Compression R in $AB - T \cos \theta = 0$.

(2) $\qquad$ $T \sin \theta - W = 0$.

Load W being given and the angle θ, equation (2) may be solved directly, and the value for T then found substituted in (1) to find R, the compression in the stick. This latter is the force with which AB *pushes to the right* against the pin B, and also *to the left* against the wall. In the model, the tension T in the tie may be checked from the reading of the balance in cord BC. The compression R may be checked by attaching a

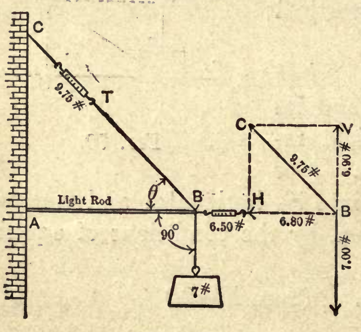


FIG. 71.

second balance at B and pulling horizontally outward until A is just free from the wall.

Example 3.—Where two of the members entering a point lie outside the X or Y axes, i.e., are neither horizontal

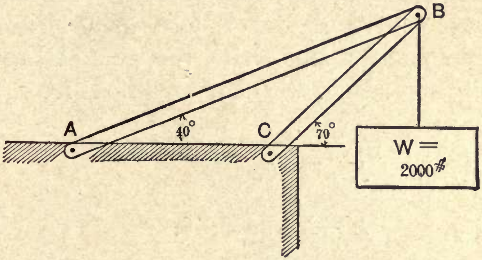


Fig. 72.

nor vertical, the student will find that his solution involves the use of simultaneous equations. These, however, have numerical coefficients and should introduce no particular difficulties. Thus, in the structure shown in Fig. 72, suppose we know necessary angles, dimensions, etc., and are given the load at B, to find the stresses in CB and AB, all joints being free to move. Consider the pin at B to be the "free body." We may then construct the force diagram shown in Fig. 73, where T represents the pull exerted by tie AB and R the thrust of stick CB.

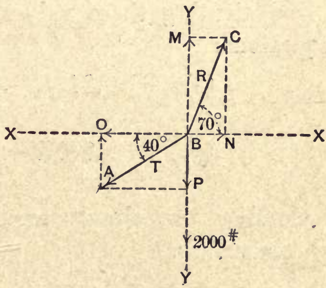


Fig. 73.

(If the student has any difficulty in understanding why T and R are given the directions they here have, let him consider: 1st. The effect upon end B of apparatus if AB

be cut; 2d. the effect if AB be replaced and then CB be cut. In what direction is each force acting upon pin B?)

This is, therefore, a case of three concurrent forces, and is capable of *graphical* solution by direct application of the parallelogram law, as in preceding problems.

Or, if we resolve R and T into their horizontal and vertical components and write the sum X and sum Y equations in accordance with our conditions for equilibrium, we shall have

$$(1) \qquad N - O = 0.$$

$$(2) \qquad M - P - 2000 = 0.$$

But
$$N = R \cos 70° = .342R,$$

$$M = R \sin 70° = .940R,$$

$$O = T \cos 40° = .766T,$$

$$P = T \sin 40° = .643T.$$

Substituting these values in (1) and (2), we have the equations:

$$(3) \qquad .342R - .766T = 0.$$

$$(4) \qquad .940R - .643T - 2000 = 0.$$

Solving the simultaneous equations (3) and (4) we obtain values,

$$T = 1370 \text{ lbs.}, \quad R = 3070 \text{ lbs.}$$

Example 4.—In the two-ton pillar crane, Fig. 74, suppose that a load of 3000 lbs. is being lifted at L. Required to find tension in BC and compression in BA, when the angles

are as indicated. Suppose the weight of jib AB is 1200 lbs., and that its center of gravity is $\frac{1}{3}$ of the way from A;

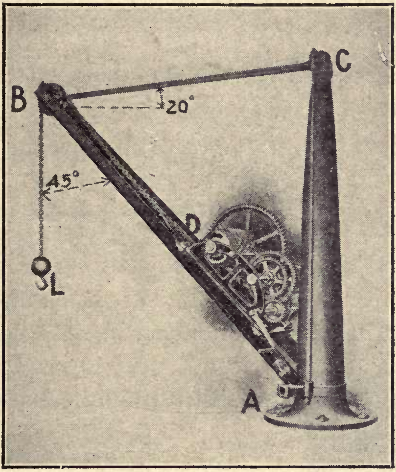


FIG. 74.—Two-ton Hand Pillar Crane.

this gives a vertical load at $B = \frac{1}{3} \times 1200 = 400$ lbs., and at $A = \frac{2}{3}$ of $1200 = 800$ lbs. The force diagram for the pin at B, taken as a free body, is therefore as in Fig. 75, the letters in the force diagram corresponding with the *members* of the crane acting at B. Friction of the pulley at B is neglected and tension in part BD of hoisting chain is assumed to be the same as tension in part BL.

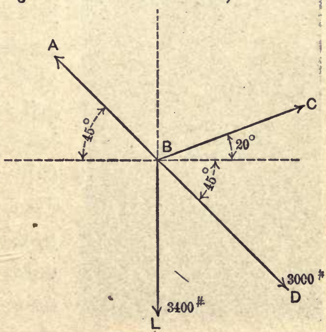


FIG. 75.

From our force diagram, using letters BC, BA, etc., to represent *forces* in the corresponding members,

(1) $BC \cos 20° + 3000 \cos 45° - BA \cos 45° = 0$. (Sum $X = 0$.)

(2) $BC \sin 20° + BA \sin 45° - 3400 - 3000 \sin 45° = 0$. (Sum $Y = 0$.)

Substituting values for the functions of the angles and transposing,

(3) $$.940BC - .707BA = -2120.$$

(4) $$.342BC + .707BA = 5520.$$

Adding equations (3) and (4),

$1.28BC = 3400$, from which tension in $BC = \frac{3400}{1.28} = 2660$ lbs.

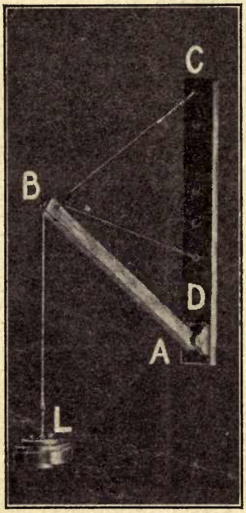


FIG. 76.—Laboratory Model Hoisting-crane.

Substituting in equation (4),

$.707BA = 5520 - .342 \times 2660$, from which compression in $BA = 6520$ lbs.

Fig. 76 shows a model of a small laboratory hoisting-crane. The angles which jib AB and cords BD and BC make with the wall may be adjusted as desired, and sufficient load applied at L to give nearly a full reading of the balance in BC. The necessary angles may then be measured, and the tension in BC and compression in BA computed, allowance being made for the weight of stick BA. The computed value of the tension in BC may then be checked by the reading of the spring balance in the cord, and the computed compression in

AB by attaching a spring balance at B and pulling in the line AB until A is just free from the pin at A.

46. Equilibrium when the Forces Act at a Common Point but do not Lie in Same Plane.—Let F_1, F_2, etc., Fig. 77, be any number of given forces acting at the point O. Through O draw three rectangular axes, X, Y, and Z. It is evident that we may resolve force F_1 into its separate effects in the directions of these axes, as OX_1, OY_1, and OZ_1. In the same way F_2 may be replaced by its components OX_2, OY_2, and OZ_2, and any other forces acting at O may be replaced by their rectangular components. We now have a new system of forces equivalent to the original forces in which all of the forces act along the axes X, Y, and Z. The resultant of all the forces along the X axis is evidently the algebraic sum of all the X components, or sum X. Similarly, the resultant of all the Y components is sum Y, and the resultant of all the Z components, sum Z. Sum X, sum Y, and sum Z are therefore equivalent to the original forces F_1, F_2, etc.

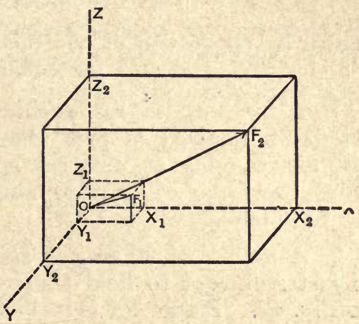


FIG. 77.

In order that forces F_1, F_2, etc., may produce equilibrium their combined effect (i.e., their resultant), must be zero. This is possible only when sum X, sum Y, and sum Z each are equal to zero. Therefore:

In order that any number of forces acting at a point, but not lying in one plane, may produce equilibrium, it is necessary that:

1. **Sum X = 0.**

2. **Sum Y = 0.**

3. **Sum Z = 0.**

PROBLEMS

(Solve by application of the conditions sum $X=0$, sum $Y=0$, for equilibrium.)

1. A force of 120 lbs. acting upward at an angle of 20° to the horizontal is just able to keep a body weighing 500 lbs. moving at uniform speed along a level surface. Find the force of friction and the pressure of the body on the plane.

2. A pull F of 20 lbs. at an angle 15° to the plane AB is just sufficient to hold the roller in an apparatus similar to Fig. 42. Plane AB is inclined at an angle of 50°. Compute the weight of roller and the pressure perpendicular to the plane.

3. A body weighing 500 lbs. suspended from a rope, is pulled aside until the suspension rope is 10° out of vertical. What will now be the tension in the supporting rope? What *horizontal* pull is required to hold the body at this angle?

4. What will be the forces in AB and BC, Fig. 72, if, in addition to the 2000 lbs. load, there is a horizontal pull to the right at B of 750 lbs.?

5. Compute the tension in BC and compression in BA of pillar crane, Fig. 74, when load L is 1000 lbs.

6. Find the forces in the members of the hoisting-crane model, Fig. 76, when $L=30$ lbs., angle $BAC=40°$, angle $CBL=120°$, and angle $CBD=50°$.

7. Compute the forces in the members of the truss, Fig. 2, for a load of 640 lbs., if the members AB and BC are inclined 30° with the horizontal and if each weighs 10 lbs. What will be vertical and horizontal reactions at foot of each stick under these conditions?

47. Non-concurrent Forces.—Forces that act along lines which are not parallel and which do not all pass through a common point are called *non-concurrent forces.*

48. Effects of Non-concurrent Forces.—A system of oblique forces acting in lines which all pass through a common point *can be replaced by a single resultant,* and will necessarily produce equilibrium *when this resultant is zero.* The *two* conditions, sum $X=0$ and sum $Y=0$, where sum X and sum Y represent the algebraic sum

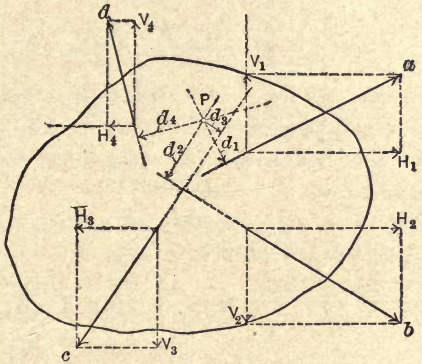


FIG. 78.

of all the horizontal and all the vertical components respectively, are therefore sufficient for equilibrium.

A system of non-concurrent forces may produce both displacement and rotation. Such a system, therefore, often cannot be replaced by a single force. A general statement of conditions, which may be applied to all systems of non-concurrent forces, must provide that both any resultant force which may produce displacement and any resultant couple which may produce rotation shall be zero.

49. Conditions of Equilibrium for Non-concurrent Forces in One Plane.—It is evident that we may resolve the

forces a, b, c, and d, Fig. 78, into horizontal and vertical components, and that if $H_1+H_2-H_3-H_4=0$, there will be no tendency for displacement in a horizontal direction; if $V_1-V_2-V_3+V_4=0$, there will be no tendency for displacement in a vertical direction.

We may also determine the perpendicular distance from each force to *any point in the plane of the forces*, as P, and compute the *moments of the forces* with respect to this point. Evidently if the clockwise moments equal the counter-clockwise, or, in other words, if

$$a\times d_1+b\times d_2-c\times d_3-d\times d_4=0$$

there will be no tendency for rotation about P. And since there is no displacement and no rotation, the forces are in equilibrium, hence we may state our conditions for equilibrium as follows:

A body acted upon by a system of non-concurrent forces in one plane will be in equilibrium when

1. **Sum X = 0;**
2. **Sum Y = 0;**
3. **Sum of all moments = 0.**

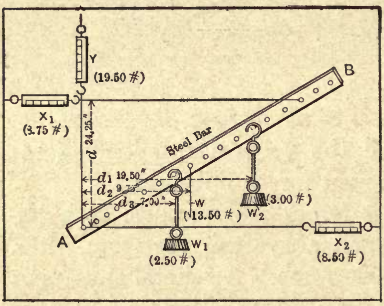


Fig. 79.—Laboratory Apparatus for Test of Conditions of Equilibrium.

Fig. 79 shows a convenient form of laboratory apparatus for testing the conditions of equilibrium for non-concurrent forces. After taking the necessary balance readings, distances, etc., and noting the application of equations (1), (2), and (3), to the apparatus as here arranged, balances X_1, X_2, and Y should be moved so that the forces do not lie in the X and Y axes

and the exercises repeated. The equation of moments should also be written for several different points along the bar assumed as the axis of rotation, and the fact noted that equation (3) is true for *any point* in the plane of the forces.

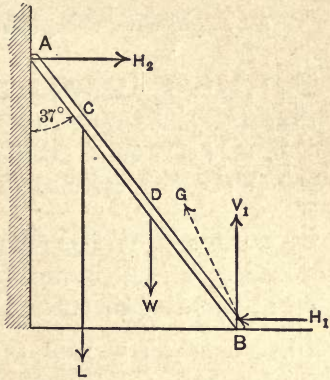


FIG. 80.

50. Solutions of Typical Cases—Forces Non-concurrent.—*Example 1.*—*AB*, Fig. 80, is a ladder leaning against a *smooth* wall making an angle of 37° with the wall. The ladder is 24 ft. long and its weight 100 lbs., acts at *D*, 10 ft. from *B*, and a load $L=300$ lbs. acts at *C*, 18 ft. from *B*. Find vertical and horizontal reactions at *B* and the horizontal reaction of the wall at *A*. (Since the wall is smooth there can be no vertical reaction at *A*.)

Equations: (1) $-H_1+H_2=0,$

(2) $V_1-W-L=0.$

Moments about *B*,

$$(3)\quad 100\times 10 \sin 37°+300\times 18 \sin 37°-H_2\times 24 \cos 37°=0;$$

$$602+3250-19.2H_2=0;$$

$$H_2=201 \text{ lbs.}$$

Therefore: $H_1=$friction, etc., to keep ladder from slipping $=201$ lbs.;

$V_1=$vertical component of reaction of ground $=400$ lbs.

The *total* ground reaction at B is, therefore,

$\sqrt{(201)^2+(400)^2}=447$ lbs. in the direction G, making an angle with the ground whose tangent is $\frac{400}{201}=2.00$.

The angle is therefore approximately 63°. (Note that the action and reaction at B is not along ladder AB. Why?)

Example 2.—In the wall crane, Fig. 81, suppose $AB=6$ ft., $AC=3$ ft., and load L is applied 2 ft. from B. Then $CB=\sqrt{36+9}=6.7$ ft. approx. Therefore $\cos\theta=\frac{6}{6.7}$, $\sin\theta=\frac{3}{6.7}$, and $d=6\sin\theta$.

The bar AB is held in equilibrium by the tension in BC, the reactions V and H at the wall and the load L. Therefore

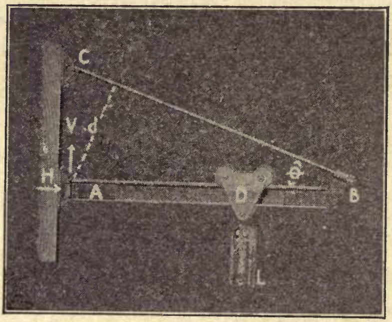


FIG. 81.—Wall or Post Crane.

(1) $H-CB\cos\theta=0$;

(2) $V+CB\sin\theta-L=0$;

(3) $CB\times 6\sin\theta-L\times 4=0$.

If load L is known, CB may be determined from equation (3). By substituting the value for CB in equations (1) and (2), the values of the hinge reactions at the wall may be found.

51. Summary of Conditions for Equilibrium.—The conditions for equilibrium under the action of the various combinations of forces in one plane may be briefly summarized as follows:

I. When two forces are acting: Forces must be equal and opposite.

II. When three forces are acting: Forces must be concurrent or parallel.

1. If concurrent:

(*a*) Graphical.—The diagonal, drawn from the point of concurrence, of the force parallelogram having any two forces as adjacent sides, must be equal and opposite to the third force.

(*b*) Algebraic.—1st. Algebraic sum of X components of all forces $=0$. 2d. Algebraic sum of Y components of all forces $=0$.

2. If parallel:

1st. Algebraic sum of forces $=0$.

2d. Algebraic sum of moments of all forces about any point in the plane of the forces $=0$.

III. When more than three forces are acting: Forces may be concurrent, parallel, or non-concurrent.

1. If concurrent:

(*a*) Graphical.—Forces must be capable of being represented to scale and in direction by the sides of a closed polygon taken in order.

(*b*) Algebraic.—As for three forces. [See II, 1 (*b*), above.]

2. If parallel:

Conditions same as with three forces.

3. If non-concurrent:

1st. Algebraic sum X components of all forces $=0$.

2d. Algebraic sum Y components of all forces $=0$.

3d. Algebraic sum moments of all forces about any point in the plane of the forces $=0$.

In general, *for any combination*, equilibrium requires that there shall be no unbalanced force to produce change in translation, and no unbalanced moment to produce change in rotation. The conditions stated under III, 3, must, therefore, be true in *all* cases. Concurrent forces are, from this point of view, a special case in which, as the forces can have but a *single resultant, if any*, the 1st and 2d conditions for non-concurrent forces are sufficient, as the 3d reduces to the form $0=0$. Likewise parallel forces are a special case in which summation along one axis only is possible.

The conditions here outlined (provided we bear in mind that we may regard a *whole structure or any selected part* of a structure as the body in equilibrium under the action of the *external* forces applied to it, and that the equation of moments may be written for *any axis* through the plane of the forces) furnish all the equations needed for the computation of the stresses in any simple structure made up of separate members pinned together at the points or with riveted joints of small area which may be assumed to be practically equivalent to pin-connections.

PROBLEMS

1. *AC*, Fig 82, is a uniform steel beam, 12 ft. long weighing 20 lbs.

(*a*) Take moments about A and thus find F_2.

(*b*) Take moments about C and thus find F_1.

(*c*) Take moments about E and thus find F_3.

(*d*) Resolve F_2 [found in (*a*)] into vertical and horizontal components and see if the three equations for non-concurrent forces are satisfied by the numerical values now found.

2. A door, Fig. 83, is hung on hinges at B and C 6 ft. apart; the door weighs 80 lbs., and its center of gravity

A is 1.5 ft. out from the line passing through the hinges. Find vertical and horizontal components at each hinge.

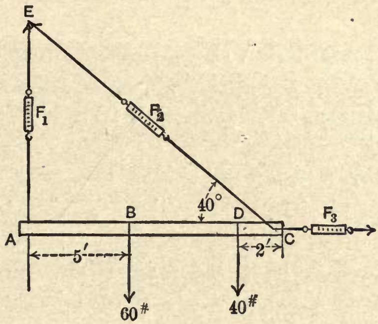


FIG. 82.

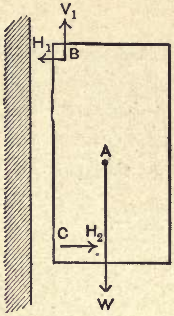


FIG. 83.

3. If friction of ladder against the building in Fig. 80 is 50 lbs., find horizontal reaction at A, and amount and direction of ground reaction at B.

4. Find tension in BC and hinge reactions for the wall crane, Fig. 81, when a load of 400 lbs. is applied successively 1 ft., 2 ft., and 4 ft. from A. [See example.]

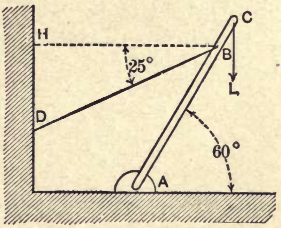


FIG. 84.

5. Find tension in BD and pin reactions at A for the pole and tie shown in the diagram, Fig. 84, when load $L = 350$ lbs., $AB = 5$ ft., $BC = 1$ ft. Weight of pole $= 80$ lbs., center of gravity $2\frac{1}{2}$ ft. from A.

CHAPTER VII

FORCES IN SOME COMMON COMMERCIAL STRUCTURES

52. Cranes and Derricks.—Fig. 85 shows a model of a derrick with guy-ropes, suitable for laboratory tests. Spring balances are inserted in the tie and guy-ropes to check the computed tensions in these members, and balances are also arranged to check the reactions at the foot of the boom. The parts are purposely made heavy so that the effect of their weight on the distribution of forces in the members may be studied, and provision is made for changing the angles of the apparatus, and the points of application of the tie and load as desired.

In the computation of stresses load L and weight W of boom, acting at its center of gravity G, are assumed as the only *known forces,* and angles and distances are measured as required. From our conditions for equilibrium, taking boom AC as the free body,

$$(1) \qquad H - BD \sin \angle BDA = 0;$$

$$(2) \qquad V + BD \cos \angle BDA - W - L = 0;$$

$$(3) \qquad BD \times d - W \times d_1 - L \times d_2 = 0.$$

These equations give the tension in BD and the reactions at A. The computed values may be checked by the balance readings. Balances V and H should be read when the foot of boom A is held just free from the mast by a horizontal pull.

At E the forces are not in one plane. We may imagine the guy-ropes EK and EF replaced by a single guy EM in the plane of EK and EF and also in the vertical plane

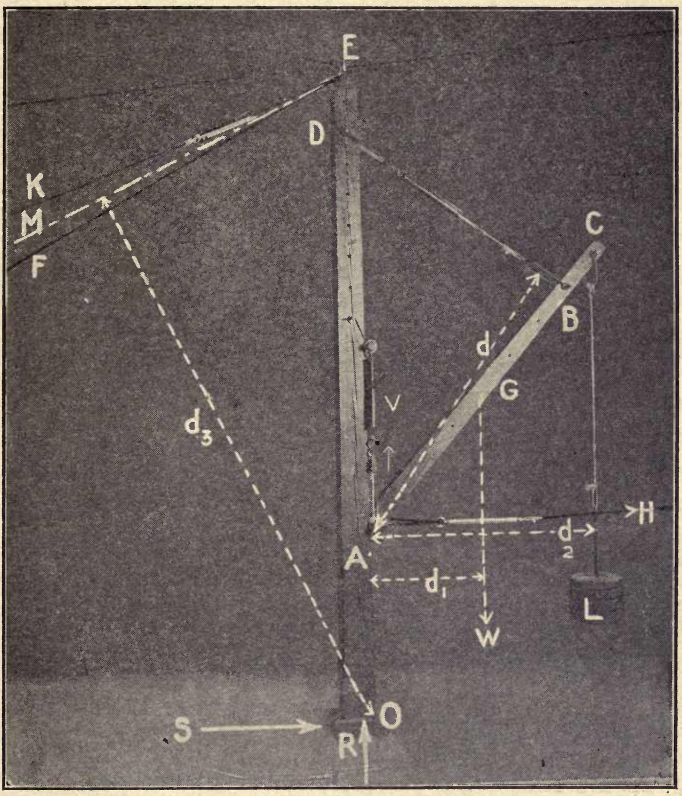


FIG. 85.—Laboratory Model Derrick and Guys.

with the axis of the mast and the load L. This single tie will support the mast, provided the boom does not swing out of the plane of the tie and mast. Therefore, taking

moments for the *whole derrick* about O, when there is no pull at A, we have the equation,

$$EM \times d_3 - W \times d_1 - L \times d_2 = 0,$$

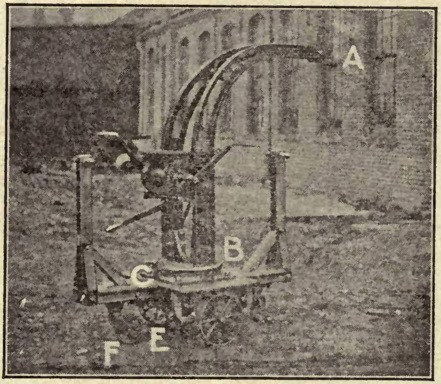


FIG. 86.—200 Pound Portable Crane.

from which the tension in the imaginary guy EM may be found. This tension may then be resolved into its components along EF and EK.

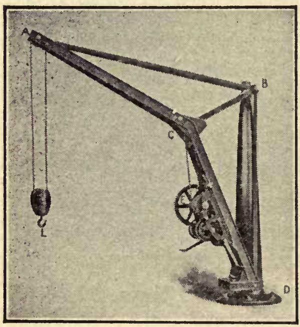


FIG. 87.—Two-ton Hand Pillar Crane.

The reaction R at the foot of the mast is obviously equal to the weight of whole crane $+L$ $+$vertical component of EM. The reaction S (when there is no horizontal pull applied at A) may be found from the equation of moments about E,

$$S \times OE - W \times d_1 - L \times d_2 = 0.$$

It is also equal to the horizontal component of tension EM. Why?

The compression in the various sections of the mast may be computed from the vertical components of the forces *above* the sections plus the weights.

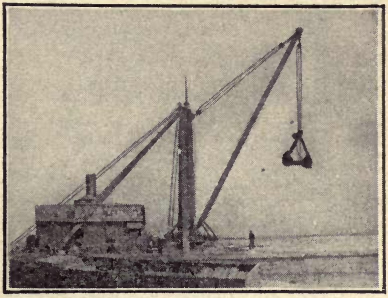


FIG. 88.—Twenty-ton Barge Derrick.

Figs. 15, 81, 86, 87, and 88 show various types of commercial cranes and derricks.

53. The Shear Legs—Fig. 89 shows a model of a pair of shears used in the physics laboratories at Pratt Institute. The legs are about 4 ft. long, of $1\frac{1}{2}$ in. oak. The "feet" are tapered and set in holes in the board MN which is screwed fast to the table. Several quick adjustments of the "spread" are thus possible. The legs are joined at A by a hinge, the pin of which passes through a clevis. By fastening a balance to the clevis and pulling up, along the line of a leg, until the foot just clears the support, the thrust at foot of the leg may be determined. The tie is of braided wire, and by an easy adjustment of its length, the legs can be

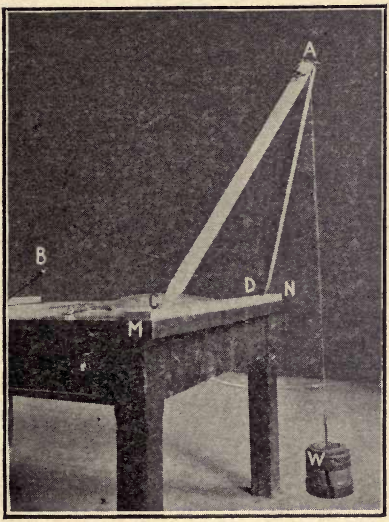


FIG. 89.—Laboratory Model Shears.

inclined at any angle to the vertical. A spring balance placed in the tie registers the tension in that member.

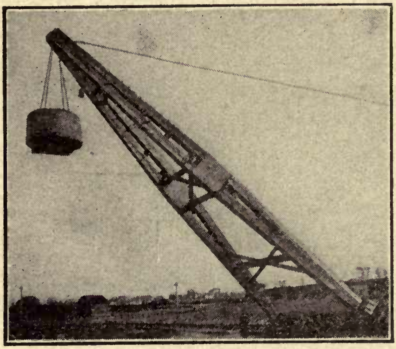


FIG. 90.—Traveling Shears.

The method of solution usually followed is as follows: About a 20-lb. load is hung at W. The legs together weigh 2 lbs., and as one-half of this may be considered as acting at A and the other half at the feet, one pound is added to the weight W in all computations.

The thrusts in the legs may be imagined to be taken up by a single member in the plane of AB and AW, running from R to A (see Fig. 92). We now have a simple case of equilibrium produced by three forces meeting at a common point. The stresses in tie AB and imaginary leg AR, are now found graphically or by trigonometry. Using the same methods, the force AR is then resolved into its two components along the legs AC and AD. By adding to these values .7 lb. (the effect of the remaining

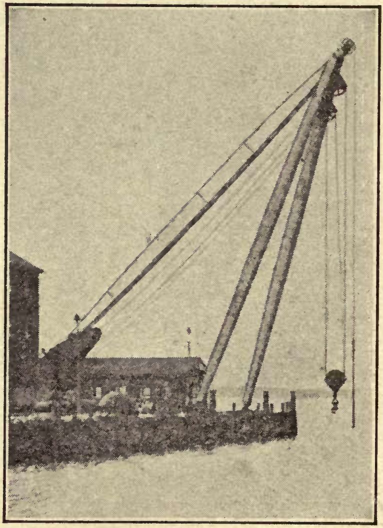


FIG. 91.—Dock Shears.

weight of the leg), the thrust at the foot is found. These results are then compared with the test readings taken as described.

A copy of the results obtained by two students, follows:

The Shears

A known weight W was hung on the model, and the angles between the different members measured. The compression in the legs and the tension in the tie AB were then computed by the following method, and the results obtained checked by reading the balances placed in the line of tie and legs:

DATA

W	18.4 lbs.
Weight of AC	1 "
Weight of AD	1 "
Angle WAB	67°
Angle WAR	25°
Angle CAR	31°
Angle DAR	31°

Method of solution:

By Fig. 92 it is seen that forces AD and AC may be replaced by a single force AR, which will be their resultant. Using this resultant together with forces AB and AW, a force diagram (Fig. 93) of the point A was drawn in which AW represented the known weight W; AB the tension in tie; AR the resultant of the thrusts in legs AB and AC.

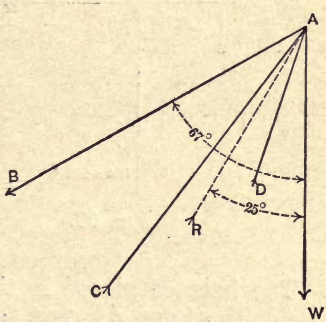


FIG. 92.

[One-half the combined weight of legs AB and AC is considered as acting at A, the other half at B and C.]

(*a*) To solve for forces AB and AR:

General law of equilibrium of forces in one plane acting at a point:

$$\text{Sum } X = 0.$$

$$\text{Sum } Y = 0.$$

Sum X.

$$(1) \qquad AR \sin 25° - AB \sin 67° = 0.$$

Sum Y.

$$(2) \qquad AR \cos 25° - AB \cos 67° - 19.4 = 0.$$

Solving (1) and (2),

$$\text{Force } AR = 26.7 \text{ lbs.},$$

$$\text{Force } AB = 12.3 \text{ lbs.}$$

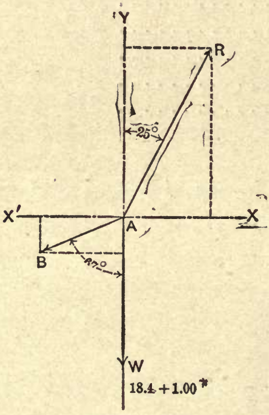


FIG. 93.

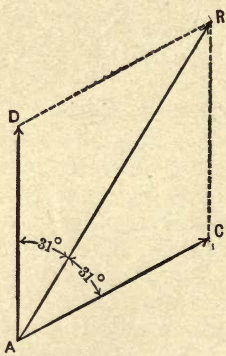


FIG. 94.

The balance placed in tie AB read 12.2 lbs.

(*b*) The force AR was then graphically resolved into its two components along the legs AC and AD by means of the parallelogram law (Fig. 94).

Compression in leg $AC = 15.5$ lbs.

Compression in leg $AD = 15.5$ lbs.

The thrust at the feet equals $15.5 + .7$ lbs. $= 16.2$ lbs.

The check reading of a balance when a pull was exerted at A to free AD at D was 16.3 lbs. The computed results vary about 1 per cent from experimental values.

SUMMARY OF RESULTS

Member.	Computed Values.	Check Readings.	Percentage of Variations.
Tie AB . .	12.3 lbs.	12.2 lbs.	1%
Leg AC . .	16.2 lbs.	16.3 lbs.	½%
Leg AD . .	16.2 lbs.	16.3 lbs.	½%

Figs. 90 and 91 show commercial shear legs, the stresses in the members of which for a given position and load may be found in the same manner.

54. Stone Tongs, etc.—The tension in the chains AB and AD of a pair of common stone tongs, Fig. 95, for a given

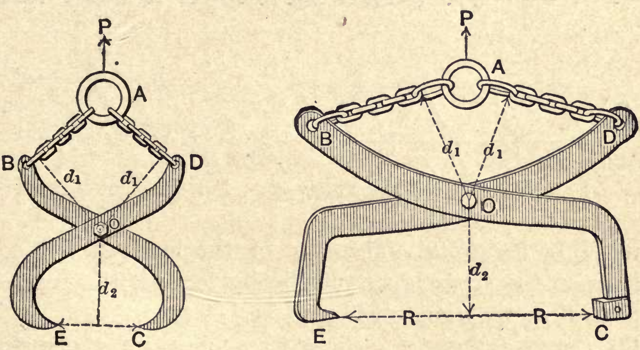


FIG. 95.—Stone Tongs.

pull P, and a given size and spread of tongs, may be obtained either graphically or algebraically, as ring A is evi-

dently held in equilibrium by three concurrent forces. The force R with which they grip the block which they support may then be computed from the equation of moments about the pin O.

55. Bridge and Roof Trusses.—Figs. 96, 97, and 98 show typical forms of simple trusses as used in bridge construction, etc. The stresses in the members of trusses of this sort may be found by taking each joint in turn as a free body and applying to each the equations for equilibrium for a system of concurrent forces. Thus, suppose the Pratt truss, Fig. 96, bears loads which are expressed in *tons* by the figures placed near the arrows. Required to find the stress in each member of the truss and to determine in each case whether the member is under tension or compression.

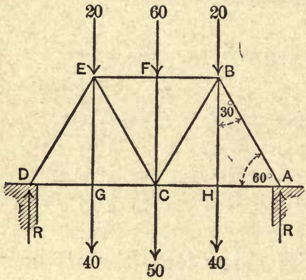


FIG. 96.—The Pratt Truss.

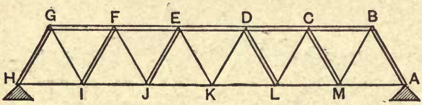


FIG. 97.—The Warren Truss.

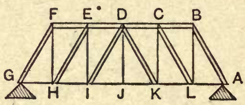


FIG. 98.—The Howe Truss.

(*a*) *Pier Reactions.*—The sum of the loads is 230 tons, and since the truss is symmetrically loaded, the reactions R and R' at the piers must each be 115 tons.

(*b*) *Forces at Point A.*—At A *three forces* act, the upward reaction R, now known to be 115 tons, a force from member BA, which from inspection we should expect to be a thrust toward A, and force from member AH which we will assume to be under tension, and therefore pulling A

toward H. Considering the joint at A as a free body, and lettering the forces in the same way as the members, we have the force diagram shown in Fig. 99. Therefore the sum X and sum Y equations are:

$$BA \cos 60° - AH = 0,$$

$$115 - BA \sin 60° = 0.$$

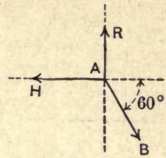


Fig. 99.

Solving, $$BA = 132.8 \text{ tons}$$

and $$AH = 66.4 \text{ tons}.$$

These equations check in every way and we may note that our first assumptions were correct, and that member AB is pushing against A (and therefore also against B), and hence is under compression; member AH is pulling on A (also on H) and is therefore under tension.

(c) *Forces at Point H.*—Next choose point H and draw its force diagram (Fig. 100). Remember if a member of any structure is under tension, *it must pull equally and in opposite directions upon the two joints* of the structure between which it is attached, or if under compression, *must push equally upon the two joints.* Every time we determine the stress in a member from a consideration of the conditions at one end, we also determine the action which it exerts upon the joint at the other end. Inspection shows that for equilibrium HC must be equal and opposite to the known tension $HA = 66.4$ tons, and also the member HB must be pulling up from H with a force equal to the 40-ton load at H.

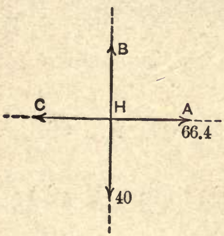


Fig. 100.

$$\therefore \quad CH = 66.4 \text{ tons (tension)},$$

$$HB = 40 \quad \text{tons (tension)}.$$

(*d*) *Forces at Point B.*—Next take point *B*. Its force diagram may be drawn as in Fig. 101. It is known from the previous solutions that the member *AB* pushes *up* and to the *left* with a force (found at point *A*) of 132.8 tons.

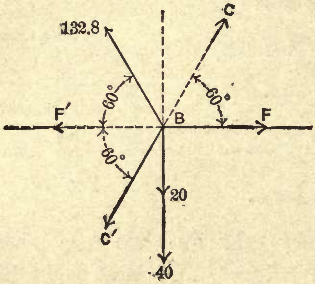


Fig. 101.

There is the downward acting load of 20 tons and also the tension of member $BH=40$ tons (see point *H*). There are two other forces acting at *B* whose amounts are as yet unknown; one from member *CB*, along the line we will call in force diagram *CC′*, and the other from member *FB* along the line *F′F*. We will call these forces *BC* and *BF*, but assume that as yet we do not know which way they act; i.e., whether members *CB* and *FB* are compression or tension members. Assume force *BC* to be upward and to the right (i.e., compression), force *BF* to be to the right (compression), and write the equations for point *B*.

$$-132.8 \cos 60+BC \cos 60+BF=0, \qquad \text{(Sum } X=0.)$$

and

$$-40-20+132.8 \sin 60+BC \sin 60=0. \quad \text{(Sum } Y=0.)$$

Solving the Sum *Y* equation we get $B=-63.5$ tons. The negative sign shows that the force along *BC* must be such as to give a negative value for its *Y* component, i.e., must be in direction *BC′*; or, in other words, *BC* must be a tension member instead of compression member as assumed.

Correcting the sign for *BC* (i.e., the direction) in our Sum *X* equation, and solving for *FB*, we obtain

$$FB=+98.2 \text{ tons.}$$

Here the positive sign shows that the member FB acts to the right at B, as we assumed in writing the equations, and BF s therefore a compression member.

(e) *Force at Point F.*—Next take point F. From symmetry we know $EF = BF = 98.2$ tons compression. The only unknown is member CF which inspection shows must be in compression and equal to 60 tons to balance

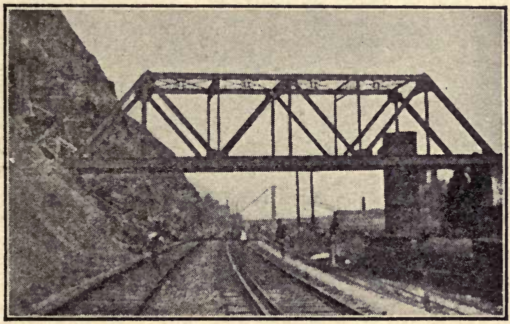


FIG. 102.—Through Span over the Wabash-Pittsburg Terminal Railway.

the load at F. Since the other half of the truss is loaded in the same way, the forces in all the members are now known, but as a check we may figure out point C.

The vertical forces are given by the equation:

$$-50 - FC + BC \cos 30 + EC \cos 30 = 0.$$

Or, $\quad -50 - 60 + 2 \times 63.5 \times .866 = 0,$ because $BC = EC$.

This reduces to $-50 - 60 + 110 = 0$, thus checking all the previous calculations.

Figs. 102 and 103 show bridge trusses in actual use, the stresses for which under given conditions of load may be determined in the same manner.

Such structures are subject to both a "dead" load, from weight of parts, ties, track, etc., and to a varying

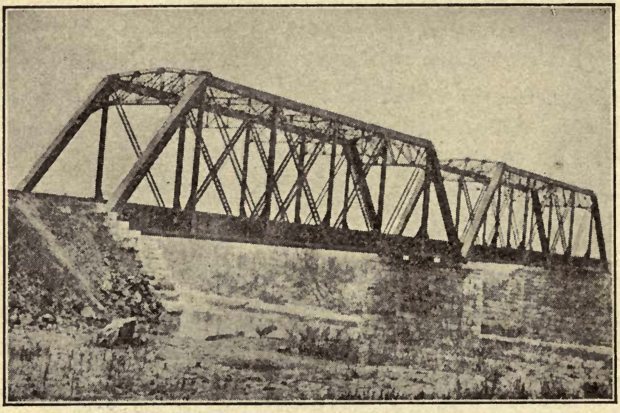


FIG. 103.—Baltimore and Ohio Railroad Bridge crossing South Branch of Potomac River in West Virginia.

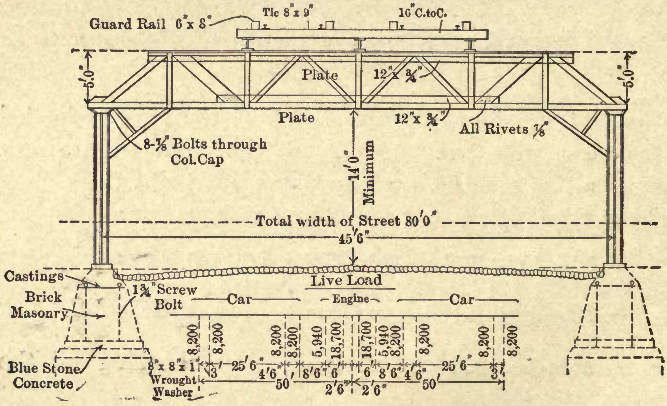


FIG. 104.—Elevated Railway, Brooklyn, N. Y.

or "live" load, coming from the weight of the engine, cars, loaded wagons, etc., hauled over them. These loads

may be applied at the joints by the method of construction (e.g., Fig. 104, which shows the manner in which the tracks of the Kings County Elevated Railway of Brooklyn are supported); if not the distributed load should be concentrated at the joints for the purposes of computation, just as in previous instances we have allowed for the weight of a part by assuming a half-weight as acting at each end.

Figs. 105, 106, and 107 illustrate typical forms of roof trusses which are also capable of solution by the "point-by-point" method. In every instance the reactions of the end supports should be determined first, and the joint at a support taken as the first point. We may then proceed from joint to joint in any order convenient so long as there are *not more than two unknown forces* at the joint considered. As we have only

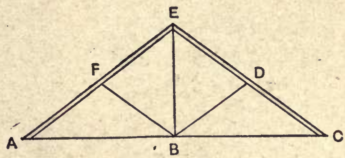


FIG. 105.

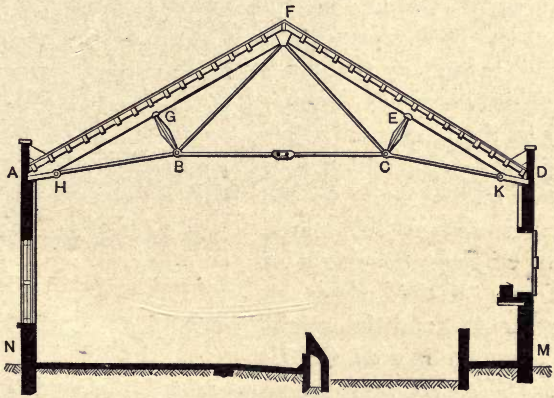


FIG. 106.—Boiler House Roof Truss.

the two equations, sum X and sum $Y=0$, only two unknowns are determinable. In trusses where these con-

ditions are not met, moments must be taken about selected points and the unknown forces reduced to two in number.

The reactions of the supports may be determined from inspection in the case of symmetrical trusses which are symmetrically loaded. In other instances, these reactions must be computed by taking moments about the supports, as for parallel forces.

The "dead" loads for roof trusses consist of the weight of the truss and the weight of the section of the roof supported. The "live" loads are the wind and snow loads. These may be applied to the joints by an auxiliary piece in the construction of the truss or in any case may be carried to the joints or apex for the purposes of computation as follows: Let shaded area *MNOP* in Fig. 108, which is a top view of a section of the roof, represent the portion of the roof and snow load taken by the truss *AEC*. This area will extend half way to the trusses on either side, and the total weight of roof and snow will be equal to the weight per square foot × area *MNOP*. Of this, the weight of the half area *MQRP* comes on the member *AE*, and the weight of the other half area *QNOR* comes on *EC*. We may therefore regard the load on *AE* as the total weight *W* of area *MQRP acting at the center of gravity of the area*, which is the *middle point of AE*, or what is the same thing, as acting ½ at *E* and ½ at *A*. The

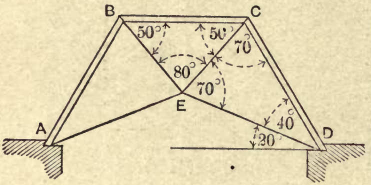


FIG. 107.

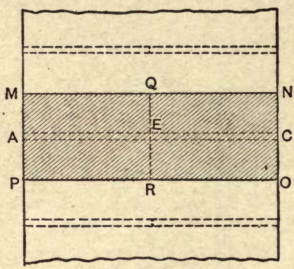


FIG. 108.

load on AE is therefore equivalent to $\frac{1}{2}W$ at E and $\frac{1}{2}W$ at A. In the same way load on EC is $\frac{1}{2}W$ at E and $\frac{1}{2}W$ at C. The total apex load at E is therefore $\frac{1}{2}W+\frac{1}{2}W=W$, and the loads at A and C are each $\frac{1}{2}W$. The student should note that the loads at A and C produce pressure on the supports but *have no effect in producing stress in members AE and EC* of the truss.

56. The Sulkey Derrick.—The solution for the forces in

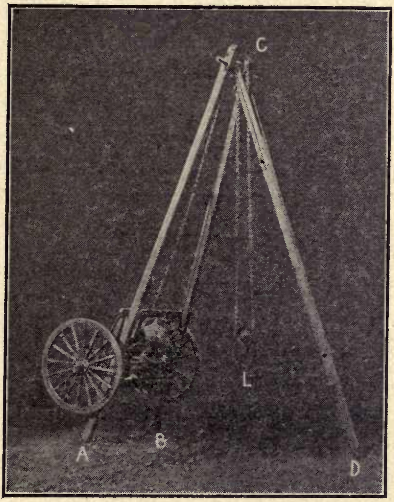


FIG. 109.—Contractor's Sulkey Derrick.

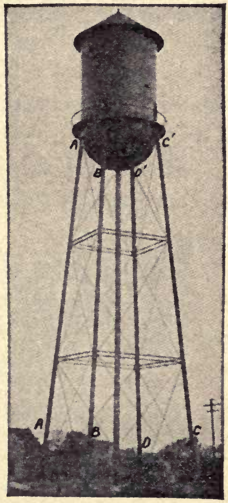


FIG. 110.—Tank City Water Works, Sycamore, Ill.

the members of a "Sulkey" derrick, Fig. 109, may be made most readily by assuming the legs CA and CB replaced by a single leg in the plane of CD and the load, as was done in the case of the shear legs. The forces at C then constitute a system of concurrent forces which may be determined in the usual way. The compression in the

single leg may then be resolved into its components along the legs CA and CB.

57. Water Tanks.—The common water tank shown in Fig. 110 furnishes another illustration of equilibrium with concurrent forces having components in three axes X, Y, and Z. A simple solution because of the ease with which dimensions, angles, etc., may be obtained is suggested in Fig. 111. Weight W of tank and contents is supported by the reactions of the four legs. If continued these five forces will intersect at a common point at O. If now we assume forces OA and OB replaced by a single force OM, and OC and OD by the single force ON, we have a system of concurrent forces in a plane. The required forces OM and ON may now be obtained graphically or algebraically in the usual way, and OM and ON may then be resolved into their components along OA and OB, and OC and OD, respectively. These are the compressions in the supporting legs.

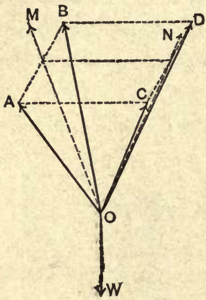


Fig. 111.

58. The Arch.—The reactions at the ends and the compression in the center of an arch may be found by regarding one half of the arch as a free body. The arrangement of forces shown in Fig. 112 then results, which is that of a simple system of non-concurrent forces. Therefore

$$(1)\quad H_2 - H_1 = 0;$$

$$(2)\quad V - W - W_1, \text{ etc.,} = 0;$$

$$(3)\quad W \times d_3 + W_1 \times d_2 - H_2 \times d = 0.$$

Loads W, W_1, etc., being known and the dimensions of

the arch, these equations may be readily solved for V and H_1, the end reactions, and H_2 the compression at the center of the arch ring. The end reactions for the other half are the same if the arch is loaded uniformly, if not similar equations may also be written for the second half. Fig. 113 shows a laboratory model of an arch in which the computed forces may be checked by spring balances. One end only of the arch is left free or "floating" at a time, the other turns freely about a pin through the horizontal frame which serves as a guide to maintain the arch in a vertical position.

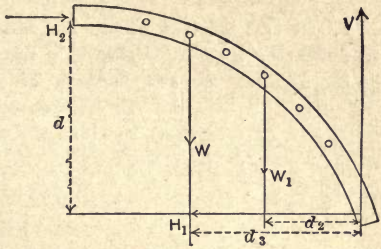


FIG. 112.

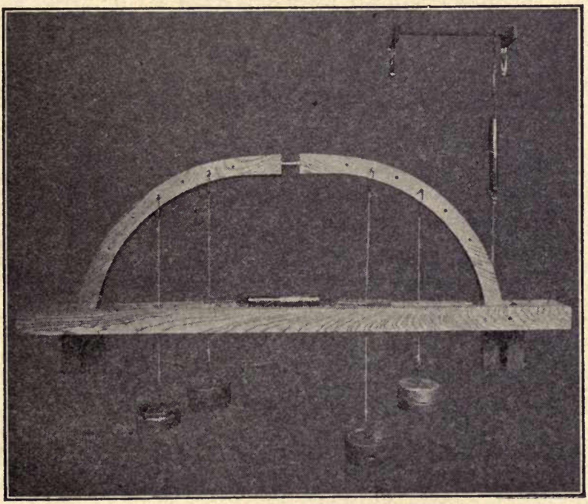


FIG. 113.—Laboratory Model Arch.

PROBLEMS

1. In the derrick, Fig. 15, mast CA is $14\frac{1}{2}$ ft. long, boom CD 20 ft. long. Guys AG and AF are each 18 ft. with 8 ft. spread at the feet. AE is 4 in. and boom weighs 320 lbs. Find tension in the tie and the guys, and thrust of boom at C, when load L is 2 tons and boom makes angle of 45° with the mast, and its vertical plane bisects the angle GAF.

2. In the portable crane, Fig. 86, center of gravity of the crane and also of the base is directly over the axle for wheels EE. Weight of crane 120 lbs., of platform 90 lbs. From vertical plane through axle to front edge B of base is 6 in., to edge C is 9 in.; to A is $3\frac{1}{2}$ ft., to front and back wheels each 2 ft. What force will be required at C when 200 lbs. are hung at A if there are no other fastenings? What will then be the pressure on each wheel?

3. In the pillar crane, Fig. 87, $AB=10$ ft., $BD=8\frac{1}{2}$ ft., $AC=7\frac{1}{2}$ ft., $CD=7\frac{1}{2}$ ft., $CB=3\frac{1}{2}$ ft., chain from C to E is vertical. Find stresses in all members when load L is 2 tons. (Neglect weight of parts and friction and note that the chain runs through block L only once.)

4. The legs of the shears, Fig. 91, are each 80 ft. in length, with a spread of 20 ft. at the base, and a *vertical height* of 60 ft. The tie and hoisting chain both reach from the top of shears to a point 40 ft. back of the middle point in the line between the feet. Find stresses in the members when supporting a load of 20 tons, if tension in hoisting chain winding on the drum is $\frac{1}{3}$ of the load.

5. Find tension in chains and force R for stone tongs, Fig. 95, when lifting a block weighing 1500 lbs., if $d_1=18$ in., $d_2=15$ in., and angle $BAD=140°$.

6. Find stresses in the members of the Howe truss, Fig. 98, when loads are 20 tons at each joint, at bottom and 5 tons for each joint at top. Angle $BAL=60°$, angle $ABL=30°$.

7. Find stresses in the members of truss shown in Fig. 102, when a truck weighing 10 tons stands on center of bridge and dead loads at each joint are $\frac{1}{2}$ ton. Angles of truss as in Fig. 96.

8. In the roof truss, Fig. 105, angles *FAB* and *ABF* are each 30°. Loads of 2 tons at *F* and *D* and 3 tons at *E*. Compute stresses in all members.

9. Find stresses due to live loads in the members of the Warren truss, Fig. 97, if each span *AM*, *ML*, etc., is 8 ft., if all angles are 60°, and if live loads are 10 tons at *K*, and 5 tons each at *J*, *L*, and *M*.

10. Find stresses in each member of the roof truss shown in Fig. 107 if loads are 5 tons each at *C* and *B* and 20 tons at *E*.

11. Find forces in the members of the sulkey derrick, Fig. 109, due to load, if the legs are each 15 ft. long, 6 in. from center to center at *C*, distance $AB = 4$ ft., distance *D* to middle of line $AB = 10$ ft., load $L = 1000$ lbs.

12. Find compression in the supports, Fig. 110, when tank contains 10,000 gallons of water, if *AB*, *BC*, etc., are each 20 ft., $A'B'$, $B'C'$, etc., each 10 ft., and vertical height of legs 60 ft. Weight of tank 5 tons.

13. Compute horizontal thrust in middle of arch ring and vertical and horizontal reactions at the piers for an arch with dimensions, and loads, as in diagram, Fig. 114.

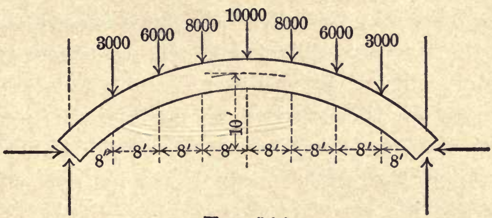


FIG. 114.

CHAPTER VIII

MOTION

BEFORE beginning this chapter, the student should review the brief discussion of motion and velocity given in Chapter II; also the application of graphical methods and the parallelogram law to motions and velocities, Chapters II and III. It should be understood that velocities, or accelerations, may be combined, resolved along axes, etc., just as we have learned to do with forces.

59. Speed. Velocity.—The rate of motion of a body is termed its *speed.*

Thus a person may travel with a *speed* of 30 miles per hour, a railroad train may increase its *speed*, etc.

The term speed expresses only *two* ideas: *distance* passed over, and *time.* It will be evident, however, from a little consideration, that these alone do not furnish sufficient data for a full consideration of a moving body. Thus, two trains, one running on a straight track, the other around a curve, may both have the same speed; the stresses to which the two are subject, and the resistance each offers to its motor, will be very different.

To account for these forces, we must know, not only the speed, but also the *direction* of the motion, and even the rate at which the direction is changing, as well as the rate at which the speed is changing. To this full description of a motion, which states the *three* specifications, *displacement*, *time*, and *direction*, the term VELOCITY is applied.

This distinction between the terms speed and velocity will be made evident by the following illustration: Suppose a railroad train starts from a given station and travels for 3 hours in a straight line at a *speed* of 30 miles per hour. From this description of the train's motion, all we know regarding its position at the end of the three hours is that it is somewhere 90 miles away on a straight line from the station. But suppose the train moves with a *velocity* of 30 miles per hour *due north.* Its position at the end of the three hours, or after the lapse of any part of the time, is now definitely fixed. A knowledge of the velocity of a body therefore enables us to determine its *position* after any time. Hence velocity may be defined as *rate of change of position.*

In speaking of the motion of bodies whose direction of motion is definitely recognized as fixed by the conditions, as, for example, a falling body, a part of a machine, a train on a given stretch of track, etc., we may use the terms speed or velocity without danger of confusion, although no actual specification of direction is given for the latter.

60. Statement of Velocity.—The velocity of a body defines its motion at *any given instant* only. Velocity may remain constant in speed and direction, or either may change from instant to instant. To express velocity, we state the *distance the body would move in a given* (or recognized) *direction in one unit of time if it maintained unchanged its present speed and direction;* thus: 30 miles (distance) per hour (time), 10 ft. per second, etc. This statement must not be interpreted as meaning that the body must actually move over the stated distance, or that motion must continue for the entire unit of time.

61. Velocity of Rotation. Angular Velocity.—The velocity of a point on a rotating body, as, for example, a point

on the rim of a fly-wheel, may be expressed in two ways: 1st, By stating the *actual length of arc swept over by the point in unit time*; or, 2d, by stating the *rate at which the point is changing its direction from the axis of rotation.* The latter expression is called the *angular velocity.*

It is evident that the first expression—i.e., the actual linear velocity of the point—depends not only upon how fast the wheel is turning, but also upon *the distance from point to axis.* Points farther out from the axis have a *greater linear velocity* than points situated nearer the center. The *angular velocity* is, however, the *same for all points* on the wheel. Angular velocity may be stated therefore for the rotating body as a whole.

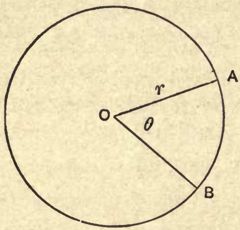


FIG. 115.

Angular velocity may be expressed in terms of degrees turned through per unit time, or, as is more customary, in terms of a special unit called a *radian.* The angle θ at the center, which intercepts an arc AB, equal in length to the radius r of the circle (Fig. 115) is called a radian. Since the circumference of a circle equals $2\pi r$, and each radian represents an arc of length r, it is evident that there are $\frac{2\pi r}{r}=2\pi$ radians in a complete circumference; and that one radian$=\frac{360}{2\pi}=57.3$ degrees approximately. The advantage of this method of expressing angular velocities is evident from the fact that *angular velocity in radians* $\times$*distance r from point to axis of rotation=linear velocity of the point.* Thus, suppose a wheel has an angular velocity of 5 radians per second; a point on this wheel 2 ft. from the axis then has a linear velocity of $5\times 2=10$ ft. per second.

In connection with machinery, velocity of rotation is

commonly expressed in "revolutions per minute." Revolutions per minute $\times 2\pi =$ radians per minute.

62. Uniform Motion. Variable Motion.—A classification of motions of great importance in mechanics is that of *uniform motion* and *non-uniform, or variable motion.* Thus, if a runner goes steadily 20 ft. every second and a proportional distance for any fraction or multiple of a second, his motion is said to be uniform; if a fly-wheel revolves steadily 200 times a minute, its motion is also uniform. But a falling stone, a train getting under way from a station, or coming to a stop, the piston of an engine, etc., are cases where the motion is not uniform.

Uniform motion is motion with a constant or unchanging velocity.

Variable motion is motion where the velocity is changing from instant to instant.

When the velocity is variable, it is, at any instant, measured by the distance through which the body *would pass in one second if it were to continue to move with the velocity it had at that particular instant.* This is called the *instantaneous velocity* of the body.

In uniform motion, whether of translation or rotation, the following relation exists:

DISTANCE = (UNIFORM) VELOCITY $\times$ TIME.

Or, in symbols, $S = u \times t.$

PROBLEMS

(See problems on graphical representation of motions and velocities, Chapter II; also on composition and resolution of motions and velocities, Chapter III.)

1. Express the velocity of rotation of the minute hand of a clock in degrees per second. In radians per second.

2. How long will it take a train to go 300 miles with a velocity of 24 ft. a second?

3. In what time will a body traverse 80 kilometers if its velocity is to be 200 cm. per second?

4. Which is the greater velocity, 300 revolutions per minute or 10 radians per second, and by how much?

5. A wheel is rotating with a uniform speed of 45 degrees per second. How many revolutions will it make in half an hour? Express its velocity in revolutions per minute. In radians per second.

6. Through what distance will a point on the wheel of Prob. 5, 2 ft. from the axis, move in 5 minutes?

7. A wheel is rotating with a velocity of 2 radians per second. Through what distance will a point on the wheel 18 inches from axis move in 1 minute?

63. Uniformly Accelerated Motion.—We have seen that variable motion is such that the velocity changes either regularly or at intervals. When the change of velocity proceeds uniformly, either increasing or decreasing, the motion is called *uniformly accelerated motion.* This is the only type of variable motion we shall need to consider.

A train starting from rest at a station and gradually getting under headway, is an illustration of accelerated motion. If the force starting the train is uniform and the resistance to motion constant, the motion of the train will be *uniformly* accelerated. The same train pulling into a station and gradually stopped by application of the brakes is an illustration of accelerated motion where the velocity is decreasing, i.e., of *negatively* accelerated or *retarded* motion. The motion of a body thrown vertically upward or falling (freely) from an elevation under the influence of gravity, is a common illustration of uniformly accelerated motion. A shaft starting to rotate under the pull of a belt, a body sliding down an inclined plane, a spinning-top

coming to rest, are all illustrations of accelerated motion; and if the forces causing and opposing motion are constant throughout the time considered, they are cases also of uniformly accelerated motion.

64. Acceleration.—Suppose a body to start from rest and, under the influence of a constant force, to move with a uniformly increasing velocity in a straight line. At the beginning the velocity is 0. Now suppose, in the case we are considering, that at the end of the first second the velocity has become 10 ft. a second; in 2 seconds, 20 ft. a second; in 3 seconds, 30 ft. a second, etc. The *gain in velocity is 10 ft. a second in each second.*

This gain of velocity in unit time is called the ACCELERATION. The body therefore starts from rest and moves with an acceleration of 10 ft/sec^2.

Or, suppose the body has a velocity of 30 ft/sec at the instant we begin to consider its motion, and that one second later its velocity is 20 ft/sec, two seconds later 10 ft/sec, and three seconds later 0 ft/sec, or body is at rest; the *loss* of velocity, or *retardation*, each second is here 10 ft/sec.

It is customary to call the CHANGE *in velocity which occurs in unit time* the acceleration, whether the change is an increase or a decrease in velocity. The distinction is merely one of direction. The acceleration is always in the direction of the force acting to produce change of velocity, but this force may be either in the direction of the original motion or opposed to it. In the first case the velocity will increase, in the second it will decrease, i.e., acceleration is negative with respect to the original velocity.

65. Acceleration of Rotating Bodies.—Acceleration in motion of rotation may be expressed in the two ways already stated for the velocity. Thus, we may state the *change in linear velocity per unit of time*, i.e., *the linear*

acceleration of a point at a known distance from the axis of rotation, or we may state the *change in angular velocity per unit of time*, i.e., the *angular acceleration.* The linear acceleration of different points in a rotating body whose velocity is changing evidently depends upon the distances of the points from the axis: the angular acceleration is the same for all points. Angular acceleration may be expressed in revolutions per unit of time gained or lost each unit (e.g., 10 revs. per min. each sec.), or as radians per second gained or lost each second.*

PROBLEMS

1. A velocity change from 0 to 50 ft/sec occurs in 10 seconds. Find the acceleration.

2. If the acceleration is 5 ft/sec^2, and the initial velocity is 6 ft/sec what will be the velocity at the end of 8 seconds?

3. In what time will the velocity of a body falling from rest become 100 ft/sec if the acceleration of gravity is 32 ft/sec^2?

4. If at 1 o'clock the velocity of a train is 15 miles per hour and at 2 minutes past 1 is 20 miles per hour, what is the acceleration in miles per hour per minute?

5. What is the acceleration in Prob. 4, expressed in miles per minute per minute?

6. A certain body moves with an acceleration of 80 ft/sec^2. What will be the change in velocity in half a minute?

7. Express the acceleration in Prob. 6 as so many feet per minute per second.

---

*NOTE: It is evident that just as angular velocity of a point expressed in radius × radians = linear velocity of the point, so also:

Angular acceleration in radians/sec^2 × radius in ft. = linear acceleration in ft/sec^2.

8. If the initial velocity of a body is 1000 ft/sec due north, and its acceleration is 50 ft/sec^2 due south, find the velocity of the body at the end of 25 seconds.

9. A ball is shot vertically upward with a velocity of 640 ft/sec, and the acceleration of gravity is 32 ft/sec^2. How long before the velocity of the body will be (*a*) 512 ft/sec upward? (*b*) Zero? (*c*) 96 ft/sec downward? (*d*) 640 ft/sec downward?

10. The velocity of a body has changed from 8 ft/sec due east to 212 ft/sec due west in 6 seconds. What has been its acceleration?

11. A wheel making 120 revolutions per minute has its velocity increased in 10 seconds to 180 revolutions per minute. Find the acceleration (*a*) in revolutions per minute gained each second; (*b*) in radians per second per second.

12. A point on a rotating body is moving 50 ft/sec. The angular velocity of the body is increased 1 revolution per minute each second for 10 seconds. What will then be the velocity of the point in ft/sec if it is situated 5 ft. from the axis of rotation?

66. Average Velocity and Distance.—Suppose a body to move with a uniform acceleration and let its velocity at the time we begin to observe its motion (initial velocity) be 6 ft/sec. At the end of 5 seconds suppose the velocity to have become 26 ft/sec. Then,

$$\text{Change of velocity } 26-6=20 \text{ ft/sec.}$$

$$\text{Acceleration}=\frac{26-6}{5}=\frac{20}{5}=4 \text{ ft/sec in one sec.}$$

The *average velocity* during the 5 seconds will be *one-half the sum of the initial and the final velocities*, or

$$\text{Average velocity}=\frac{6+26}{2}=16 \text{ ft/sec.}$$

The *total distance traversed will be the average velocity multiplied by the time,* or

$$\text{Distance} = 16 \times 5 = 80 \text{ ft.}$$

It is of the utmost importance for the student to master thoroughly these simple relations. They are stated formally as follows, and apply equally to rotation and translation:

Change of velocity = Acceleration × time;

Final velocity = Initial velocity + change of velocity;

Average velocity = ½ (initial velocity + final velocity);

Distance = Average velocity × time.

NOTE: It should be noticed that these statements are *perfectly general for all cases* of uniformly accelerated motion. Thus, if a body start from rest, its initial velocity is 0. Also, if the initial velocity is in one direction and the acceleration is in the *opposite* direction, the rules above still apply if we give to one direction the + sign and to the other the − sign.

ILLUSTRATION.—A body is moving N. with a velocity of 40 ft/sec, and a force acts S. so as to give it an acceleration S of 8 ft/sec^2. What will be its velocity at the end of 7 seconds, and how far from the start will it be at the end of that time? Call the initial velocity +40 and the acceleration −8. Then

$$\text{Change of velocity} = -8 \times 7 = -56;$$

$$\text{Final velocity} = +40 - 56 = -16;$$

$$\text{Average velocity} = \frac{40-16}{2} = 12;$$

$$\text{Distance } 12 \times 7 = 84 \text{ ft.}$$

That is, the body will, at the end of the 7 seconds, be 84 ft. N. of its original position, and will be moving S. with a velocity of 16 ft/sec.

(What has actually happened if the full history of the motion is required, is that the body has moved N. until brought to rest, then has started S. from zero velocity and moved until its velocity is 16 ft/sec S.

To lose all velocity N. will require $\frac{40}{8}=5$ seconds, and the distance moved N. will be $\frac{40+0}{2}\times 5=100$ ft.

The time it moved south is therefore $7-5=2$ seconds. Velocity at the end of the 7th second is therefore $2\times 8=16$ ft/sec S. Distance moved S. after stopping $=\frac{0+16}{2}\times 2$ $=16$ ft. Therefore the body is $100-16=84$ ft. N. of starting point.)

67. Equations for Accelerated Motion.—To express the relations of the various items of velocity, time, distance, etc., in accelerated motion by simple equations, we may make use of the following notations:

$u=$ initial velocity;

$v=$ final velocity;

$t=$ time;

$a=$ acceleration;

$S=$ distance.

Then $\quad a\,t=$ change in velocity,

and $\quad v=u+a\,t.$

Also, $\quad$ Average vel. $=\frac{u+v}{2}$ or $\frac{2u+a\,t}{2}=u+\frac{1}{2}a\,t$

and $\quad$ Distance $=(u+\frac{1}{2}a\,t)\times t=u\,t+\frac{1}{2}a\,t^2.$

68. Accelerated Motion from Rest.—If a body starts from rest, that is, if $u=0$, u disappears from the preceding equations and $S=\frac{1}{2}a\,t^2$.

The distance passed over by a body starting from rest and moving with a uniform acceleration, therefore *varies as the* SQUARE OF THE TIME. Hence. if the body moves

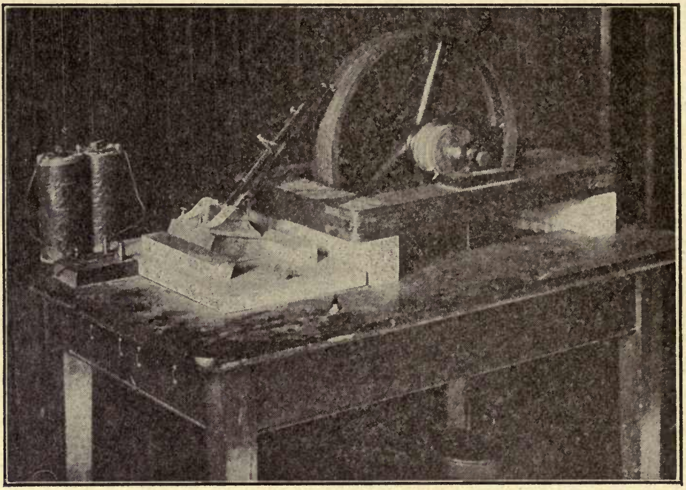


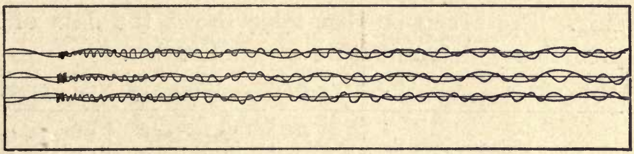
FIG. 116.—Apparatus for Study of Accelerated Rotary Motion.

10 ft. the first second, it will move $2\times2\times10=40$ ft. in 2 seconds, $3\times3\times10=90$ ft. in 3 seconds, etc.

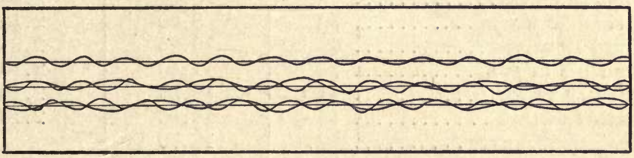
The student should test the preceding statements for motion of translation by use of the Atwood's machine, body moving down an incline, or any of the well known apparatus for studying the laws of accelerated motion.

Fig. 116 shows a convenient device for studying accelerated motion of rotation. The moving body consists of a pulley P, accurately turned and centered, mounted

upon hardened cone bearings. Motion is caused by a weight W descending and unwinding a cord from the drum C. To obtain a time record a strip of paper, such as is supplied in rolls for adding machines, is placed tightly around the pulley and the overlapping ends pasted together. The paper is then smoked until evenly coated with soot. As the pulley is rotated, the vibrations of an electrically-driven tuning fork are traced upon the paper.



Section at Beginning of Trace.



Section of Middle of Trace.

FIG. 117.—Specimen Tracing of Tuning Fork.

In use, the fork is first placed on the paper and by a transverse motion a straight line is drawn to mark the zero from which to measure distances. The wheel is then released and the trace taken for about two revolutions, when the fork is raised and the battery switch opened. Several records may be obtained, side by side, by shifting the fork. The paper is then cut and fixed by passing through thin shellac. A specimen tracing is shown in Fig. 117.

The first two or three waves of the tracing are not legible as all the soot is removed. To locate the 10th

wave, a preliminary determination of the acceleration for a 10-wave period is made; the 10-wave period is then used throughout as the unit of time. In this preliminary determination an arbitrary point of reference, R, a couple of centimeters from the starting line of the trace, is chosen, and the distances passed over in successive periods of 10 waves is measured from this point. The gain in velocity per 10 waves, in a 10-wave period is found then by finding the gain in distance of each 10-wave period over the preceding. The accompanying table shows the data of such a preliminary determination of acceleration.

Distance from R.		Distance in 10 Waves.	Distance Gained in 10 Waves.
First 10 waves	3.99	6.11	2.09
Second 10 waves	10.10	8.20	2.15
Third 10 waves	18.30	10.35	2.05
Fourth 10 waves	28.65	12.40	2.10
Fifth 10 waves	41.05	14.50	2.10
Sixth 10 waves	55.55	16.50	2.10
Seventh 10 waves	72.15	18.70	
Eighth 10 waves	90.85		

From the table it is seen that the *average* gain in velocity per unit time (10 waves), which takes place in each 10 waves period, is 2.10 cms. The gain in speed during the *first unit of time* when the trace is illegible is therefore also 2.10 cms.; and as the initial velocity was zero, the average velocity must have been $\frac{0+2.10}{2}$, or 1.05 cm. per unit time. This distance may now be measured off from the starting point and *marked as the first 10-wave point.* By counting, the succeeding 10-wave distances are marked. Measurements are then continued from the 10-wave point

thus fixed, giving the distances passed over in the actual time elapsed since the release of the wheel or *true zero* of time. From this record of time and distance curves may be plotted as in Fig. 118, verifying the laws of accelerated motion of rotation, i.e.,

$$\text{Velocity} \propto \text{time}$$

and

$$\text{Distance} \propto (\text{time})^2.$$

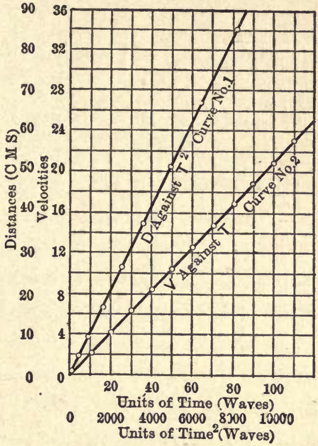


FIG. 118.

Since the distances on the rim of the wheel are proportional to the *angular* displacement corresponding, the laws of *angular acceleration* under a *constant torque*, are seen to be exactly analogous to the laws of *accelerated motion in a straight line* under the action of a *constant force.*

69. Motion of a Body under the Influence of Gravity. A body falling under the influence of gravity, if we neglect the resistance offered by the air, furnishes a common instance of uniformly accelerated motion.

In this course, we shall always assume the acceleration produced by gravity on all bodies to be

$$32 \text{ ft. per sec}^2,$$

or

$$980 \text{ cm. per sec}^2.$$

This value is represented by the symbol g. The equations for falling bodies are therefore,

(1) $S=ut+\frac{1}{2}gt^2$ for bodies having initial velocity.

(2) $S=\frac{1}{2}gt^2$ for bodies falling freely from rest.

Since $$v=gt \quad \text{and} \quad t=\frac{v}{g},$$

Substituting in (2) we have $v=\sqrt{2gS}$ as the expression for the velocity acquired by a body in falling freely from rest through the distance S.

The value of the force of gravity and therefore of the acceleration that it produces, g, varies somewhat from place to place on the earth's surface, being, in general, least at the equator and increasing towards the poles. It is also greater at the surface of the earth than at points below or above the surface. These variations, however, are small and for most purposes the values 32 ft/sec^2 and 980 cm/sec^2 are sufficiently exact for our use.

70. Solution for a Body Projected Vertically Upward or Downward.—Suppose a ball is projected vertically upward with a velocity of 192 ft/sec. Find: (*a*) The time during which it will rise; (*b*) the height to which it will rise; (*c*) its velocity after 8 seconds; (*d*) its velocity on striking the ground.

(*a*) $$\text{Initial velocity} = -192,$$
$$\text{Acceleration} = 32.$$

Therefore, since its velocity at the highest point of its journey (final velocity) is 0, we have

$$0=-192+32t,$$

$$t=6 \text{ seconds.}$$

(*b*) Initial velocity $= -192$;

Final velocity $= 0$;

Aver. velocity $= -96$;

Distance $= -96 \times 6 = -576$ ft., or body will rise 576 ft.

(*c*) Change of velocity in 8 sec. $= 32 \times 8 = 256$;

Final velocity $= -192 + 256 = +64$ ft/sec. Or body will be moving 64 ft/sec *downward.*

(*d*) It took 6 seconds to reach the highest point where the velocity was 0. Considering now only the return journey, we may call the

Initial velocity $= 0$;

Acceleration $= 32$;

Final velocity $= 32t$ (where t is the time of return journey.)

Aver. velocity $= 16t$;

Distance $= 576$ ft.;

or
$$576 = 16t \times t = 16t^2,$$
$$t = 6,$$

and the velocity on reaching the ground will be $6 \times 32 = 192$ ft/sec.

Thus we see that, in this case, the time of rise will be equal to the time of the return. The whole time of flight is 12 seconds. Also the downward velocity on reaching the ground will be numerically equal to the upward velocity with which the body started.

In the case of a body projected vertically downward we may apply the laws of accelerated motion in the same way, the initial velocity and the acceleration of gravity being here in the same direction.

PROBLEMS

1. A sled moves from rest until in 12 seconds it has acquired a velocity of 60 ft. a second. Find the acceleration, the average velocity, the velocity at the end of 7 seconds, and the distance traversed in the 12 seconds.

2. If an engine takes 24 seconds to get up a velocity of 300 revolutions per minute in its fly-wheel, find (*a*) the angular acceleration, (*b*) the angular velocity at the end of 15 seconds, (*c*) the average angular velocity during the process of speeding up.

3. The acceleration of a body starting from rest is 3 ft/sec^2. Plotting times and velocities for the end of each of the first 5 seconds, get the time-velocity diagram and draw on it the line representing the average velocity. Determine also the distance. Plot time as abscissæ.

4. For 6 seconds after starting, a body has a positive acceleration of 3 ft/sec^2. It then proceeds with the velocity already acquired. Draw the time-velocity diagram with the final velocities for the first 10 seconds. (Time as abscissæ.)

5. A bicycle and rider start from rest at the top of a hill. At end of 10 seconds the foot of the hill is reached with a speed of 3 feet per second. Find the acceleration (considered as uniform) of the wheel during the descent.

6. A body going with a uniform speed of 50 ft. per second is acted upon by a force which gives it an acceleration per second of 5 ft. per second. How fast will the body be going at the end of 30 seconds?

7. A body is going 28 ft. per second, when its velocity is increased 4 ft. per second each second for 6 seconds. Find the velocity at the end and the distance traveled.

8. The velocity of a body changes from 400 ft. per second to 180 ft. per second in 5 seconds. Find the acceleration and the distance.

9. An acceleration of 30 (feet and seconds) equals what acceleration in feet and minutes?

10. A railroad train starts from rest at a station and increases its speed uniformly at the rate of $\frac{1}{2}$ ft. per second each second. How fast will it be going at the end of 20 seconds?

11. Express the speed of Problem 10 in terms of miles per hour.

12. A body going at the rate of 500 ft. per second has its speed uniformly accelerated until at the end of 10 minutes its speed is 1000 ft. per second. Find the acceleration per minute in feet per second.

13. A body going 100 ft. per second has its velocity decreased to $\frac{1}{2}$ ft. per second in going 100 ft. Find the acceleration.

14. A train running at rate of 30 miles per hour is stopped by a sudden application of the brake. What negative acceleration must be given in order that the train may be brought to rest (at a uniform rate) in 20 seconds?

15. A wheel starting from rest has its speed increased uniformly each second 15 degrees per second. What will be its velocity of rotation in degrees per second at the end of one-half minute?

16. If the wheel in the preceding problem were already making 10 revolutions per second when the increase began, what would be its velocity at the end of one-half minute?

17. A car is moving uniformly at the rate of 3 ft. per second. What uniform acceleration must be given it so that at end of 10 seconds it may be going at speed of 30 ft. per second?

18. An elevator starting from rest moves with an acceleration of 1 ft/sec^2 for ten seconds. Find final velocity, average velocity, and distance.

1 foot-per second per second.

19. A railroad train, going 30 miles per hour, increases its speed uniformly each second, $\frac{1}{2}$ ft. per second for 5 minutes. At what rate will it then be moving (in miles per hour) and how far will it have gone during the 5 minutes?

20. A moving body has its velocity changed uniformly from 1500 ft. per second to 300 ft. per second in 30 seconds. Find the negative acceleration. How far will it go meantime?

21. A car sliding down a grade changes its velocity from 20 ft. per second to 4 ft. per second. If this is done with a uniform negative acceleration per second of 2 ft. per second, what time elapses during the change? How far does the car go meanwhile?

22. Find the acceleration of a body which starting from rest acquires a velocity of 300 cms. per second while moving through a distance of 1200 centimeters.

23. A body starting from rest increases its speed each second 5 ft. per second. How far will it move during the fourth second?

24. A wheel making 5 revolutions per second has its velocity of rotation increased each second 30 degrees per second. How many revolutions per second will it be making at the end of 10 seconds?

25. A street car has its speed diminished from 40 ft. per second to 15 ft. per second. If the brakes were applied with such force as to diminish the car's velocity each second 5 ft. per second, how far did the car go in the meantime?

26. To what height and for what time will a body rise if projected vertically upward with a velocity of 100 ft. a second?

27. With what initial velocity must a body start in order to rise vertically 700 ft. before losing all its velocity?

28. What initial velocity must a body have if it is to move with a positive acceleration of 4 ft/sec^2 for 13 seconds, and pass over 500 ft.?

29. A body going 150 ft. a second has its speed decreased to 50 ft. per second in going 240 ft. Find the acceleration.

30. A ball is thrown vertically downward from the top of a tower 550 ft. high. In 5 seconds it is 25 ft. from the ground. What was its initial velocity and when will it reach the ground?

31. A ball is dropped from a certain point and $\frac{1}{4}$ of a second afterwards another ball is dropped from the same point. How far apart will they be when the second ball has fallen 2 seconds?

32. A balloon is rising at the rate of 20 ft. a second when a sand bag is dropped from it. In what time and with what velocity will the bag reach the ground if the balloon was half a mile high when the bag was dropped?

33. A stone thrown vertically upward from the ground under an arch is seen to just touch the arch and return to the ground in 4.5 seconds after it started. How high is the arch?

34. Find the acceleration if a train starting with a velocity of 15 miles per hour passes over 350 ft. in 10 seconds.

35. With what velocty must a body be projected vertically upward to attain a height of 6000 ft.?

36. A wheel starting from rest is given an acceleration each second of 3.4 radians per second. Through what distance will a point 2 ft. from the axis of rotation move in 20 seconds?

37. The velocity of a pair of rollers in a paper mill changes from 80 revolutions per minute to 120 revolutions per minute, in 30 seconds. If the paper is passed between them without slippage, what length of paper will be passed in 30 seconds, assuming the diameter of the roll as 20 inches?

71. Path of Bodies Projected Horizontally or at an Angle of Elevation less than 90°.—1. *Motion of a body projected from a height with a uniform horizontal velocity.* If a body is projected from a height in a horizontal direction, the velocity of projection and the acceleration of gravity are at right angles to each other and the body will move in a curve. The position of the body at any time, if we disregard air resistance, may be determined by remembering that the horizontal velocity will in no way affect the rate of vertical fall under the influence of gravity, nor will the fact that the body is falling in any way affect the uniform horizontal velocity with which it started. This is really an illustration of the truth of "Newton's Second Law of Motion." Thus, if a body be projected horizontally from the top of a tower with a velocity of 400 ft. a second, it will move in 3 seconds a distance of $3 \times 400 = 1200$ ft. out from the tower. In the same time it will of course have fallen with an acceleration of 32 ft/sec^2.

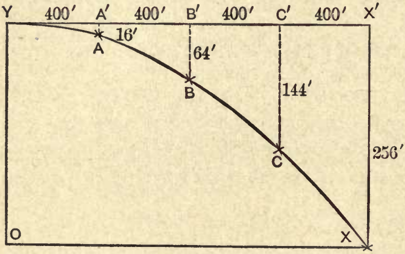


Fig. 119.

Its average vertical velocity will be $\frac{1}{2}(32 \times 3) = 48$ ft. per second, and the vertical distance fallen $48 \times 3 = 144$ ft. Then at the end of the 3 seconds, if we neglect modifications of its motion due to the air through which it has passed, it will be 144 ft. below the top of the tower and 1200 ft. out from it.

If the positions of a body under the conditions just described are calculated for several intervals of time and the results plotted, a curve, as in Fig. 119, will be obtained

showing the position of the body at every instant, and the general shape of its path. This curve is a *parabola*.

2. *Motion of a body starting with a uniform velocity in a direction inclined to the horizontal at less than 90 degrees.* Suppose a body projected from O (Fig. 120) along the line OE, making an angle d with the horizontal. Let OA represent the velocity with which the body is projected from O. Then if gravity did not act, the body in one second would be at A, in 2 seconds at B, in 3 seconds at C, in 4 seconds at D, etc., where $OA = AB = BC$. But since gravity also acts, the body will be *below* A at the end of one second, *by an amount equal to the distance it will fall in one second*, or it will be 16 ft. vertically below A. Similarly, at the end of 2 seconds it will be 64 ft. vertically below B, etc. In this way we may construct a curve giving the path of the body when we have given the amount and the angle of the original velocity.

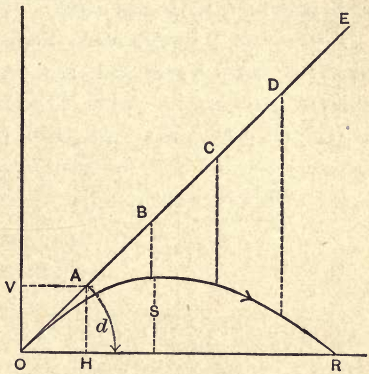


FIG. 120.

The greatest height, S, and "range," OR, of this curve may be computed as follows:

Resolve OA into its component horizontal and vertical velocities, H and V.

$$H = OA \cos d \quad \text{and} \quad V = OA \sin d,$$

from which the numerical values of H and V may be calculated if OA is given. Now the horizontal and the vertical motions may be considered separately. The

component V is the *initial vertical* velocity and H is the *uniform horizontal* velocity. Thus, $\frac{V}{32}=t$, the time during which body will continue to rise. Therefore,

$$S=\left(\frac{V+0}{2}\right)\times t.$$

Since the body will require the same time to fall as was required in rising, the whole time of flight will be $2t$.

Therefore "horizontal range," $OR=H\times 2t$. No allowance is here made for air resistance, wind, etc. These effects are variable with the shape of the projected body, initial velocity, etc., and special corrections must be applied for any actual case of projection, as, for example, in the firing of long-range guns.

PROBLEMS

1. A stone is thrown horizontally out from the top of a cliff 248 ft. high with a velocity of 70 ft. per second. Draw a diagram to scale showing starting point, point of fall, and four intermediate positions.

2. When and where will a ball strike the ground if it is thrown horizontally out from the top of a tower 200 ft. high with a velocity of 20 ft. per second?

3. A body has simultaneously a uniform velocity due north of 40 ft. per second, and an acceleration due east of 20 ft/sec^2. Find its position at the end of the first, second, third, fourth, and fifth seconds.

4. Plot the path of the body in Problem 3, and from the curve take the position of the body at the end of 3.6 seconds.

5. A projectile is fired at an angle of 30 degrees to the horizontal with a velocity of 144 ft. per second. Draw a diagram of its path showing its position at the end of each of the first 5 seconds.

6. Find the greatest height reached and the horizontal range of a projectile the velocity of which is 450 ft. per second at an angle of 60 degrees to the horizontal.

7. A projectile is thrown with an initial velocity of 600 cm. per second at an angle of 50 degrees with the horizontal. Find total time of flight and the horizontal range.

72. Simple Harmonic Motion.—A third type of motion, as distinguished from uniform motion and uniformly accelerated motion, is illustrated in the periodic swinging back and forth between two extreme positions of a pendulum-bob, of the free end of a flexible rod when clamped at one end and then pulled aside and suddenly released, of a weight suspended from a spring, etc. Motion of this sort is known as *simple harmonic motion.* All vibrations due to the elastic properties of bodies are simple harmonic. The to-and-fro motion of the cross-head of a steam-engine is approximately simple harmonic.

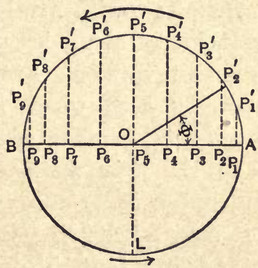


FIG. 121.

The general characteristics of simple harmonic motion are shown in the following illustration:

Imagine a point P' to move with a *uniform velocity* v in a circular path (Fig. 121), and imagine a perpendicular dropped from each successive position of this point to a diameter AB of the circular path. As P' moves around the circle, point P, the intersection of this perpendicular with the diameter will move periodically back and forth between the two extreme positions A and B. The time required for P to move from A to B and then back again from B to A, is called the *period* of P's oscillation. It is evidently the time required for P' to travel once around its circular path.

Inspection of the figure shows also that while P' *moves with uniform velocity*, passing over the equal arcs, AP'_1, $P'_1P'_2$, etc., in successive equal intervals of time, point P travels in the corresponding time intervals over distances AP_1, P_1P_2, etc., which *are unequal* and which *increase as P moves toward the center O*, and *decrease as P moves away* from the center. The velocity of P along the diameter at its extreme positions A and B, when P' is moving for an instant at right angles to the diameter, is zero. At P'_5 and the corresponding position on the other semicircumference, P' is moving for an instant parallel to the diameter AB, hence at these points, P, which is passing through O, moves with a maximum velocity which is equal to the velocity of P', or v. Starting from an extreme position as A, therefore, with a velocity of zero, P moves with increasing velocity along its path to O where its velocity has a maximum value v. From O to B, the velocity of P *decreases*, becoming zero when B is reached. Similar changes in velocity take place as P moves back over its path from B to A. *The motion of P is therefore accelerated, the acceleration being always toward the center.*

It may be shown also that the *value of P's acceleration is not uniform, but is dependent upon the position of P on the diameter AB.* An approximate demonstration of this important idea may be made graphically as follows: Draw Fig. 121 to a large scale and measure the distances AP_1, P_1P_2, etc. The data given in the table opposite was obtained in this way.

If we assume the time required for P to move over each space as AP_1, P_1P_2, etc., as "unit" time, since this time is very short, we may regard the distances AP_1, P_1P_2, etc., measured along the diameter as representing the average velocities for each successive unit of time. The values in column one of the table were obtained in this way. The

distance P_1P_2—the distance AP_1 therefore represents the gain in velocity in unit time or in other words, *equals the acceleration at* P_1. Similarly, distance P_2P_3 −distance P_1P_2 represents the acceleration at P_2, etc. In this way values were found for the accelerations given in column two. The "displacement" is the distance from the center, O, to the point under consideration. Thus the displacement at P_1 is represented by the distance OP_1, at P_2 by the distance OP_2, etc.

Average Velocity.		Acceleration.		Displacement.		$\frac{\text{Displacement}}{\text{Acceleration}}$.
For Space.	Value.	At.	Value.	At.	Value.	
AP_1	.60					
P_1P_2	1.72	P_1	1.12	P_1	11.39	10.2
P_2P_3	2.64	P_2	.92	P_2	9.64	10.4
P_3P_4	3.33	P_3	.69	P_3	7.03	10.2
P_4O	3.70	P_4	.37	P_4	3.70	10.0
OP_6	3.70	O	....	O		
P_6P_7	3.33	P_6	.37	P_6	3.70	10.0
$P\,P_8$	2.64	P_7	.69	P_7	7.03	10.2
$P\,P$	1.72	P_8	.92	P_8	9.64	10.4
$P\,B$	.60	P_9	1.12	P_9	11.39	10.2

From the values in the table it will be seen that the velocity of P increases from A to O and then decreases from O to B; and that the acceleration is zero at O and a maximum at points A and B, also that the direction of the acceleration is always toward the center O. The values for the fraction displacement over acceleration given in the last column of the table are practically constant, thus showing that acceleration is proportional to displacement.

And, finally, if we substitute the average value for displacement over acceleration in the equation,

$$T=2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}},$$

and solve for T, we have the approximate value 20, which we see, since each space represents unit time, is the time required for a complete oscillation of P along the diameter.

A body having a motion similar to that of point P in the preceding illustration is said to have *simple harmonic motion.* The characteristics of simple harmonic motion are:

1. It is in the nature of a periodic oscillation back and forth between two extreme positions.

2. The velocity of the moving body is zero at its positions of greatest displacement from the center of its path, and a maximum when passing in either direction through the center.

3. The body is constantly accelerated toward the center of its path.

4. The acceleration is not uniform but is dependent upon the position of the body in its path; the law of variation being: "Acceleration is proportional to the displacement."

5. The period of a simple harmonic motion is expressed by the equation,

$$T = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}}.$$

73. Other Applications of Laws of Simple Harmonic Motion.—Many quantities which undergo periodic changes in value do so in accordance with general laws similar to those of simple harmonic motion. An important case of this nature is furnished by the variation of pressure at the brushes of an alternating current generator. The recurring cycle of values through which such voltage passes, together with its periodic reversal in direction, follows laws approximately identical with those governing the velocity of the point P in the illustration just discussed. The *rate of change* of voltage is not uniform but undergoes changes similar to

the changes in the value of the acceleration of the point P, the governing factor being the position of the generating coil in its revolution.

74. Phase of a Simple Harmonic Motion.—Since both the determining factors of a motion, velocity, and acceleration are dependent in simple harmonic motion upon the position of the moving body in its path, it is convenient to have some easy way of fixing the stage arrived at in the oscillation. Thus while point P in our illustration, Fig. 121, is the body actually moving with a harmonic motion, it is convenient to project P back upon an imaginary circle and fix its position and therefore all conditions of the harmonic motion by locating point P' on the "circle of reference." The position of the imaginary radius vector OP', assumed to start from the reference line OA (see Fig. 121) and to rotate in the direction of the arrows, then shows clearly the stage which has been reached in the simple harmonic motion. The angle ϕ between this radius vector and OA, is known as the *Phase* of the simple harmonic motion.

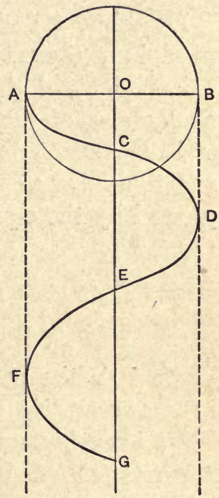


Fig. 122.

75. The Sine Curve.—If the point P, moving as in Fig. 121, were able to make a tracing upon a surface held under the circle, and if this surface were moved uniformly in a direction at right angles to AB, the result would be a curve, as $ACDEFG$ (Fig. 122). This curve, which is characteristic of simple harmonic motion, is called a *Sine Curve*, since the ordinates of points on the curve are proportional to the sines of the corresponding angles of phase.

If we imagine the abscissæ of this curve to represent time, or some corresponding quantity, the sine curve represents clearly the successive stages in the simple harmonic motion. Curve $CDEFG$ represents a complete *cycle* of changes. Sine curves are commonly used in representing periodic changes in an alternating current, alternating current voltages, etc.

76. Pendulums.—A simple pendulum consists of a ball suspended by a thread so light and flexible that its mass and stiffness may be disregarded. Such a pendulum furnishes an illustration of simple harmonic motion, for if set vibrating, the ball will pass periodically back and forth over its path, with a maximum velocity at the middle point of its swing, and zero velocity at the ends where the direction of motion reverses.

Experiment and theory both show that the *period* of a pendulum (i.e., the time for a complete oscillation) is independent of the mass of the pendulum-bob and practically also of the length of arc through which it swings, and dependent only upon the length of the pendulum and the value of gravity. The law of the pendulum is expressed by the equation,

$$T=2\pi\sqrt{\frac{l}{g}},$$

where T is the period, l the length of the pendulum, and g the acceleration due to gravity at the locality.

77. Experimental Tests of the Laws of the Pendulum.—The relation of the period of a pendulum to length may readily be shown experimentally by determining the period of a pendulum consisting of a metal ball suspended by means of a silk thread, when the length of the pendulum (measured from support to center of ball) is successively

1600, 900, 400, and 100 millimeters long. The ratio of the square roots of the lengths is then 40:30:20:10 or 4:3:2:1. The corresponding periods will be found to be practically 4:3:2:1, or the "period of a pendulum varies as the square root of the length."

Fig. 123 shows a form of pendulum which may be used to demonstrate the relation of period to force acting. This pendulum may be inclined at any desired angle to the vertical and its period then determined. Angles with the vertical are measured by means of a vernier attachment upon the base *C*. The bearings of the pendulum are steel cones giving little friction.

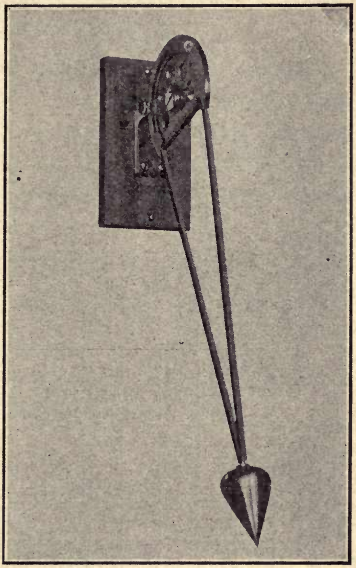


Fig. 123.—Inclined Pendulum for proving relation of Period of Vibration to Force Acting.

When the axis of the pendulum is inclined at an angle θ with the vertical, the force of gravity G is resolved into two components, a and b, Fig. 124. One of these, a, is counteracted by the bearings. The other, b, causes the pendulum to oscillate.

It is evident from the figure that $b = G \cos\theta$; therefore by changing the angle, different values of the force causing oscillation may be obtained. Thus, when the pendulum is vertical, $a=0$, $b=G$; at an angle $\theta=30°$, $b=G\cos 30° = .866\,G$, etc. Assuming $G=1$, a series of *relative* values for the component b may be obtained. Plotting these

results with corresponding periods as ordinates will give curves as *A* and *B*, Fig. 125. It is evident from the straight line *B* that the period varies as $\frac{1}{\sqrt{\text{force}}}$. This law, which may also be stated, "The square of the number of vibrations varies directly as the force acting," is a general law for periodic vibration. An important illustration is seen in the case of a magnetic needle oscillating in a magnetic field. The period of such a pendulum will depend upon the

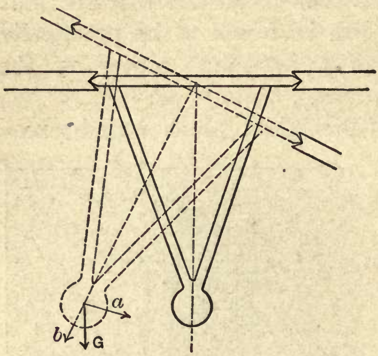


FIG. 124.

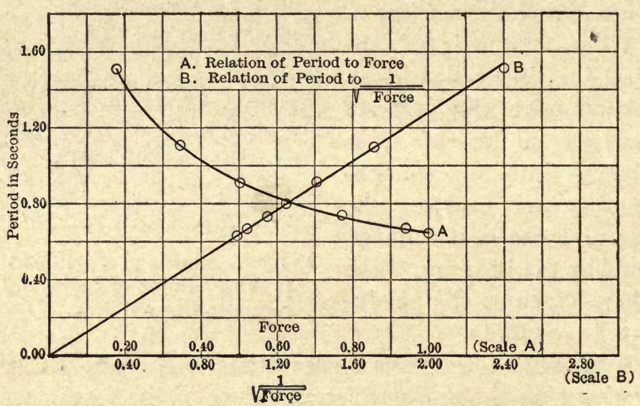


FIG. 125.

intensity of field in which it is placed, or $n^2 \propto H$ where H represents intensity of field.

78. Determination of Acceleration Due to Gravity.—The acceleration of a freely falling body differs slightly for different localities. Various methods have been used for determining its value, the most common being some form of pendulum. If a simple pendulum of known length is oscillated and its period found, the values for length and period may be substituted in the equation for the pendulum and the expression solved for "*g*." Various special forms of the pendulum, designed to give more accurate determinations of period or of length, have been devised for this experiment. A description of Borda's and Kater's forms will be found in almost any laboratory manual of physics. A device which has the advantage of being simple and easily manipulated, and which does not require that the speed of the falling body be lessened by a counterweight, as in the Atwood's machine, or by using only a component of the force of gravitation, as on an inclined plane, is shown in Fig. 126.

The pendulum, by means of which both the time and the distance are determined, consists of an oak rod about 9 ft. long swinging on a knife-edge at C. The steel plates on which the knife-edge rests are carried on a platform standing on leveling screws M, by means of which the pendulum may be caused to swing in a true vertical plane. The upper end of the pendulum is composed of an iron rod GH passing through the knife-edge to which it is held by the thumb-screw N. This allows for the adjustment of the length and therefore of the period of the pendulum within certain limits. Through a slot near the upper end of the pendulum projects a slender arm RS carrying the electromagnet T, capable of holding a small steel ball. Near the bottom is a metal hook K which engages with a metal strip on the end of a lever L so fastened as to hold the pendulum about 10 or 15 degrees from the

vertical. By raising this lever the pendulum is started and the electric circuit through the magnet is broken at the same instant, therefore the pendulum and the ball begin their motions simultaneously.

The magnet is set in such a position that the ball would fall exactly along the face of the pendulum, if the latter were at rest in its vertical position, and hence, if ball and pendulum start together, they will strike when the pendulum has made exactly half a swing. It is evident therefore, that the time of fall can be found by determining the period of the pendulum. Where the ball strikes it leaves a mark about an eighth of an inch long on a chalked piece of black vulcanite carried on the face of the pendulum. The distance from the electromagnet to this mark is the fall corresponding to a half vibration.

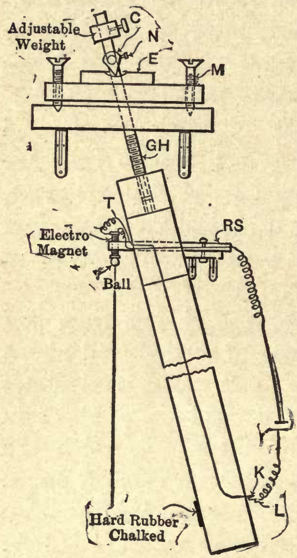


FIG. 126.

The arrangement of the apparatus is such as to allow all the necessary adjustments to be made quite readily and exactly. By using a good stop-watch and timing a long series of vibrations, the period of the pendulum can be found very closely. The chief source of difficulty is in determining the exact point where the ball strikes the chalked surface. As may be seen from the data given below, these points vary in position by not more than 2 cms. in about 250, which is considerably less than one per cent, while in most cases the difference is only a few millimeters. This close agreement would seem to

indicate also that the error to be anticipated from the fact that the ball in falling will not necessarily pass over an exactly vertical path, thus meeting the pendulum at times a little short of its middle position, at others a little beyond, is small. Thus by averaging a number of readings, a probable value may be found which is not only quite close enough for the purposes for which the apparatus is designed but which approaches very closely the true value.

The following table shows the results which the apparatus is capable of giving:

Distance of Fall, S.	Average, S.	Period of Pendulum.	t.	t^2.
248.2				
247.8				
248.2				
249.0				
248.0				
248.5	247.7	1.424	.712	.507
247.4				
246.5				
247.0				
247.0				

Therefore, by the general forumla, $S=\frac{1}{2}at^2$, we obtain $g=977$ cms/sec^2, a result within about one-third per cent of the accepted value 980 cms/sec^2. The value 977 here obtained is, of course, that of a body falling in air instead of in a vacuum.

PROBLEMS

1. If a pendulum of length 30 in. has a period .885 second, what is the period of one 20 in. long?

2. If the period of a certain pendulum is 1.2 seconds, what is the period of one three times as long?

3. A pendulum whose length is 6 makes 40 vibrations per minute. How many will one whose length is 2 make in the same time?

4. A certain pendulum is found to make one vibration in .8 second at a place where the value of g is 31.8. What will be its period at a place where g is 32?

5. Calculate the length of a pendulum whose period is 2 seconds.

6. The period of a pendulum 95 cms. long is 1.06 seconds. From these figures find the value of g.

7. The rope carrying the car of a mine shaft is 900 ft. long. What will be the period of swing as a pendulum?

8. A rigid pendulum is inclined so as to make an angle of 40° with the vertical. If its period when hanging vertical was .51 second, what will it be when thus inclined?

9. A rigid pendulum makes 2 vibrations per second when hanging vertical. How many will it make when inclined at an angle of 50° to the vertical?

10. Compute the length of a seconds pendulum (one having a period of one second), at a place where g is 980.3 cms.

11. A clock having a pendulum 40 cms. long keeps correct time. The pendulum is lengthened to 40.2 cms. How many seconds will the pendulum lose in one day?

CHAPTER IX

FORCES PRODUCING MOTION

It has been stated that a force may produce *either or both* of two effects upon a given body: (*a*) Change in its state of motion; (*b*) change of form. In our study of forces thus far, we have assumed a "*rigid body,*" i.e., one in which any change of form which may occur is too slight to materially affect the particular conditions with which we were concerned, and have confined our attention solely to those cases in which the forces were so balanced that the body considered was *maintained unchanged in its existing state* of *motion*, i.e., *continued at rest or in uniform motion.* For this, we found it necessary that for every force which would produce displacement in any given direction there should be an exactly equal force in the opposite direction, and for every torque which would cause the body to rotate in one direction about a given axis, there should be an equal torque about the same axis, but in the opposite direction.

In this chapter we again assume a rigid body, but have to deal with those cases in which the force in any direction is not exactly neutralized, and therefore change of motion occurs in the direction of the unbalanced force; or in which the torque is not balanced and therefore the rotating body increases or decreases its speed. In other words, we have to consider all those cases in which a body is *acquiring velocity, is coming to rest, or is changing the*

direction of its motion. We are to investigate the laws which govern such changes, and are to determine the equation which shall enable us to compute the force required to increase or decrease the speed of a given body by a stated amount in a stated time, or the time required for a force to produce or destroy a given velocity.

79. Meaning of the Term Mass.—It is a matter of common experience that it is more difficult (i.e., requires more force) to set in motion a loaded car than an empty one, to stop a heavy ball rolling along than to stop a lighter one moving with the same velocity, etc. To take an ideal case not capable of perfect experimental proof, consider two blocks of the same shape and size, one of wood and one of lead, resting on a perfectly *smooth* horizontal table. Attach two coiled springs exactly alike, one to each block, and pull horizontally on each so that the two springs will be extended exactly equal amounts. Each block will begin to move in the direction of the applied force, but the block of wood will move faster than the block of lead, i.e., will have a *greater acceleration.* Here equal forces act on two bodies under identical conditions. Why are the resulting accelerations unequal? Because the lead has more matter in it, or to use the exact term, the lead has a greater *Mass.*

In general, it is a result of our experience in attempting to set bodies in motion, that where there is more *matter* to be moved there will always be less motion, i.e., less velocity acquired in a given time under the action of the same amount of force. This may be put more formally by saying that under the influence of a given force, a body's acceleration is less in proportion as the amount of matter in the body is greater.

It is this fact which we recognize by saying that *matter has inertia.*

Definition.—The amount of matter in a body is called its *Mass.* Thus, in the illustration of the blocks of wood and lead just given, we say that the lead block has a smaller acceleration because it has a greater *mass.*

The inertia of a body exhibited when an attempt is made to change its velocity, has already been described as one of the forms of reaction against which applied forces act.

80. Relation between Force and Mass.—The general relation of the force required to produce motion to the quantity of matter moved, and the rate at which velocity is to be acquired, was suggested in the preceding article.

The exact law of this relation ship may be illustrated by the following simple experiment:

W is a light wheel over which weights *A* and *B* are hung by a light flexible cord. (Fig. 127.) We will assume that there is no friction and that the masses of wheel and cord are so small that they may be neglected. Let *A* and *B* be brass blocks of the same size and shape, each 45 grams, and let *R* be a third block of 10 grams. When released *B* and *R* will descend, and *A* will ascend with accelerating motion, and by observing the distance the system moves in a given number of seconds we may calculate the acceleration.

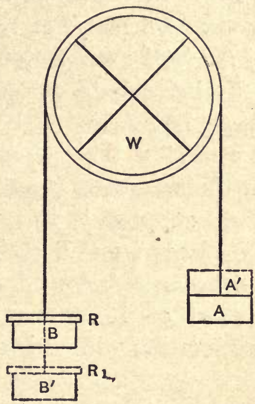


FIG. 127.

Suppose the system moves 196 cms. in 2 seconds. The average velocity is $\frac{196}{2}=98$; the final velocity $98\times 2=196$, and the acceleration is 98.

Now add masses of 45 grams to each side. *The acceleration will be less*, and a second extra mass R' must be put on in order that acceleration may again be 98.

We may tabulate our facts as follows:

	Force.	Total Mass.	Accel.
Case 1	10	100	98
2	10	190	less than 98
3	20	200	98

From our data we see that *in order to produce the same acceleration the force required must be greater in proportion as the total mass moved is greater.*

To investigate our problem still further we may proceed thus: Let A and B be equal, and each 45 grams, and let R consist of three equal pieces each 5 grams. Put all three on the side with B and find by test as before the acceleration; suppose it is found to be 140. Now remove *one* of the pieces from B and put it on A. The mass moved is the same as before, but the moving force is now 5 grams instead of 15 grams. We shall find that the resulting acceleration will be less—will be, in fact, 46.6 or $\frac{1}{3}$ of 140.

	Force.	Total Mass.	Accel.
Case 1	15	105	140
2	5	105	46.6

We have therefore found that the acceleration given to a certain body depends upon the forces applied, *and that if the force acting is increased the acceleration will be increased in the same proportion.*

To summarize the results of our experiment in a more condensed and mathematical form, we may use the following symbols: f = force, m = mass, a = acceleration.

From the first part of our experiment, *for a constant acceleration, force varies as mass*, or

$$f \propto m.$$

From the secona part, *for the same mass, the acceleration produced varies as the force applied*, or

$$f \propto a.$$

Therefore, $$f \propto ma.$$

This is a perfectly general expression of the *natural law* which applies whenever a body is set in motion or when the motion of a moving body is modified in any way.

81. Relation of Certain Units of Mass, Force, and Acceleration. So far we have shown merely that forces are proportional to the masses moved, and the accelerations produced, or, in general, $f \propto ma$. But this expression in its present form does not enable us to determine *the amount of force, measured in recognizable force units,* that will be required to impart a desired acceleration to a particular mass. Before such computation is possible, the expression must be changed to the equational form, $f = ma$, or "force *equals* mass × acceleration;" and before such an equation can be written, we must choose units of force and mass which will make the equation *true numerically*.

In our common English units the *unit mass* is called the pound. The *force* of gravity drawing this mass toward the earth is also named a pound. It is thus evident that we use the term *pound* in two quite distinct senses, the first indicating a *mass*, that is, a quantity of matter exhibiting a certain inertia, the second indicating a *force*. Now,

if we let a pound *mass* fall it is acted on by a *force* of a pound, and we know that the resulting acceleration is $g=32$ ft/sec in one second. Thus it is *not true* that

Force (in pounds) = (mass pounds) × acceleration in feet and seconds, would for this give $1=1\times 32$. In other words, we may not use the expression pound *in two different senses in the same equation.*

But suppose we elect to use the unit, *pound*, as a weight or *force unit*, and then adopt such a unit of mass that our equation will be numerically true. Such a mass unit will evidently be the mass of a body weighing approximately 32 pounds because the weight of a body gives to its mass an acceleration of approximately *32 ft/sec*^{2}. In other words, we will use a mass unit which will satisfy our equation if we divide the weight of a body in pounds by 32, and our equation may now be written,

$$1 \text{ lb. (force)} = \frac{\text{lbs. weight}}{32} \text{ (mass)} \times \text{acceleration in ft/sec}^2,$$

which is true.

If, therefore, we express the force F acting upon a body in gravity units or pounds, the acceleration a with which the body is caused to move in feet per second in one second, and the mass of the body in terms of its weight W in pounds divided by g (i.e., 32 ft/sec^2), we may express the true relation between the three quantities by the general equation,

$$F=\frac{W}{g}a.$$

This is the equation commonly used by the engineer who finds it more convenient to use the pound as a unit of force, and who therefore derives a unit of mass to satisfy the equation $f=ma$, by expressing mass in terms of $\dfrac{\text{weight}}{g}$.

If we use F to mean the force in grams, W the weight in grams, and g the acceleration in centimeters per second in one second (980), we have the same equation again. a must, of course, always be expressed in the same units as g.

Illustrations.—(*a*) What acceleration will result if a pull of 80 pounds acts on a body weighing 640 pounds? Here $F=80$, $W=640$, and

$$80=\frac{640}{32}\times a,$$

whence $$a=4 \text{ ft/sec}^2.$$

(*b*) A body weighing 2940 grams is acted on by a constant force and an acceleration of 8 results. Find the force.

$$F=\frac{2940}{980}\times 8,$$

$$F=24 \text{ grams}.$$

These two units of force, the pound force and the gram force, are sometimes called *Gravitational Units of Force.*

82. The Dyne.—Assume a unit of force just sufficient to give a mass of one gram an acceleration of 1 cm/sec². Evidently this is less force than the force of gravity on the gram mass, for the latter would give it an acceleration of 980 cms/sec². But adopt this force as a *new unit* of force, then in this case,

f (force) $=1$, m (mass) $=1$, and a (acceleration) $=1$, and we may write,

$$f=ma,$$

an equation numerically correct for these units by definition. This new unit of force is called a *Dyne.*

Definition.—A dyne is the force which, acting on a mass of 1 gram, will give it a velocity of 1 cm. per second in 1 second.

The dyne is seen to be $\frac{1}{980}$ of a force of 1 gram, or more strictly,

$$1 \text{ gram}=g \text{ dynes}.$$

Illustrations.—(*a*) What force acting on a mass of 800 grams will in 5 seconds give it a velocity of 600 cms. per second?

The acceleration $=\frac{600}{5}=120$ cms/sec^2.

$$f=800\times 120.$$

$$=96{,}000 \text{ dynes}.$$

(*b*) If a force of 10,000 dynes acts on a mass of 200 grams for 3 seconds, what velocity will it produce?

$$10{,}000=200\times a,$$

$$a=50 \text{ cms/sec}^2,$$

$$50\times 3=150 \text{ cms. per second}.$$

(*c*) With what force in dynes is a mass of 60 grams drawn toward the earth at a place where $g=980.5$?

The force is 60 grams (weight),

$$1 \text{ gram} = 980.5 \text{ dynes},$$

$$60 \times 980.5 = 58{,}830 \text{ dynes}.$$

(*d*) A force of 90,000 dynes is found to produce an acceleration = 1800 on a certain mass. Find the mass.

$$90{,}000 = 1800m,$$

$$m = 50 \text{ grams}.$$

In a similar manner we might choose a force which, acting on a mass of 1 pound, would give it an acceleration of 1 ft/sec^2. This force would be only $\frac{1}{32}$ of a force of one pound.

Calling this new unit of force by a new name, *Poundal*,* we again have the equation,

$$f = ma,$$

where f = poundals, m = pounds mass, and a = acceleration in feet and seconds.

The dyne and poundal are called *Absolute Units*, because their values do not depend on the value of g, which varies from place to place on the earth's surface and at points below or above the surface.

83. Mass and Weight.—The foregoing discussion suggests the distinction between the *mass* of a body and its *weight*.

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* NOTE.—We shall seldom have occasion to use *poundals*, since in all engineering work with the English units forces are more conveniently expressed in pounds. The *dyne*, however, is frequently used in practical mechanics, especially in electrical and chemical calculations.

Mass means quantity of matter measured by its inertia.

Weight means strictly the force due to gravity with which the body is attracted to the earth.

Since the quantity of matter does not change when a body is moved from one place to another, the *mass is constant.* But the weight may change, indeed, usually does change. It is less at the equator than near the poles, for instance. Also on the moon the weight of what we call a pound of iron would be much smaller than on the earth, but its mass would be the same, for the same force applied to it would produce the same acceleration.

At the center of the earth a body would have the same mass as elsewhere but no weight at all, but if a mass were in motion there, the same force would be required to stop it as at the earth's surface.

84. Summary.—The essential meaning of the term *Force* is that which gives or tends to give accelerated motion to a mass, and it is measured by the product of mass and acceleration. If a body has no acceleration (is in equilibrium) it does not necessarily follow that there is no force; it may mean that there are two equal and opposite forces acting, or any system of forces such that they neutralize each other. Any *one of the forces alone* would produce motion, the reaction being in the inertia of the body. Two systems of force measurement are in common use. In one the terms pound, gram, etc., are taken as names of *force units.* In this system mass is expressed by the derived unit, weight, This system is called the gravitational system. In the other system the pound, gram, etc., are *units of mass.* A unit of force is then derived by assuming as unit force that force which will impart unit acceleration to unit mass.

The table below shows the relations between units.

Force Units.	Mass Units.	Accel. Unit, a.	Equations.	
			Gravitation.	Absolute.
F=pounds	$\frac{W\text{(lbs.)}}{g}=\frac{W}{32}$	1 ft/sec.^{2}	$F=\frac{W}{32}a$	
f=poundals	m=lbs.	"	....	$f=ma$
F=grams	$\frac{W\text{(gm.)}}{g}=\frac{W}{980}$	1 cm/sec.^{2}	$F=\frac{W}{980}a$	
f=dynes	m=gm.	"	....	$f=ma$

85. Motion of a Body on a Smooth Incline.—In Fig. 128 OA represents the total pull of gravity W on a body O. Then OB and OC represent the components of W parallel to and perpendicular to the face of the incline. If δ is the angle between the plane and the horizontal then $OB=W\sin\delta$, or

$$OB=W\times\frac{RT}{SR}.$$

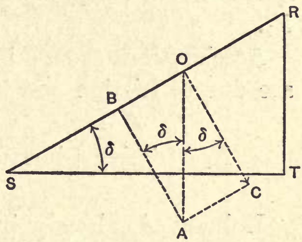

FIG. 128.

If the surface SR is smooth (i.e., if there is no friction), the component OC does not affect the motion, and the moving force is OB tending to give motion down the plane. Since the full force of gravity W will give an acceleration g, the force $OB=W\times\frac{RT}{SR}$ will give an acceleration $a=g\times\frac{RT}{SR}=g\sin\delta$.

86. Acceleration Diagrams and the Solution of Problems.—When a body is made to move with accelerated motion in any direction, the force F_1 in the direction of motion must always be greater than the force, F_2, opposing motion.

The difference, $F_1 - F_2$, represents the *unbalanced force causing acceleration.* Opposed to this unbalanced force is an *equal reaction due to inertia,* or $F_1 - F_2 =$ inertia reaction $= \dfrac{\text{weight of body or bodies moved}}{g} \times$ acceleration. If we include this inertia reaction as an *equivalent force opposing change of motion,* we may write the general equation for accelerated motion:

Sum (*all actions in the directions of the change of velocity*)
$=$ *Sum* (*all reactions in the opposite direction*).

Thus suppose a body A is made to slide with increasing speed over a rough horizontal surface, Fig. 129, by a pull F_1.

Acceleration occurs in the direction of F_1; reactions are due to friction and inertia. Therefore

$$F_1 = \text{friction} + \text{inertia reaction}\left(\text{i.e., } \frac{\text{wt. } A}{g} \times \text{acc.}\right).$$

If body A, already in motion, is being brought to rest by friction, our "acceleration diagram"* will be as in Fig. 130. Friction is decreasing the velocity (i.e., pro-

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*Note: The student must not confuse what is here called the "acceleration diagram" with the force diagram of a free body in equilibrium. The moving body is not in equilibrium. In reality, the force F, which causes A to move (Fig. 129), comes from some second body B. *The inertia reaction of A is therefore really* exerted *on B and not on body A*, as shown. Fig. 129 *is* therefore *not a force diagram for body A*. The student will, however, find such "acceleration diagrams" helpful in more complicated cases of accelerated motion, because they help to create a clear idea of the true conditions, and tend to prevent the omission of any forces or reactions affecting the motion of the given body.

ducing a negative acceleration), and the inertia of A is tending to continue its motion. Therefore,

$$\text{Friction} = \text{inertia reaction} = \frac{\text{wt. } A}{g} \times \text{acc.}$$

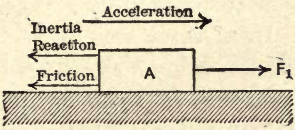


FIG. 129.

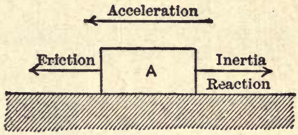


FIG. 130.

In general, in the solution of problems, proceed as follows:

1. Determine the direction of the change in velocity.

2. Construct an "acceleration diagram," showing all actions in the direction of the change in velocity, and all reactions in the opposite directions. *Always include among the latter, one due to inertia.*

3. Substitute the values for these actions and reactions in the general equation and solve.

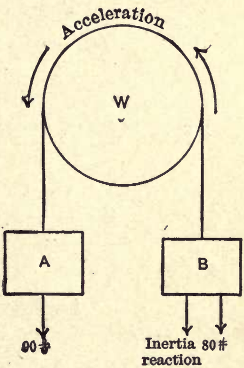


FIG. 131.

ILLUSTRATIONS. *Example 1.*—Weights of 90 and 80 pounds hang by a cord over a pulley (Fig. 131). Find the acceleration of the system, if friction and the mass of cord and wheel may be neglected.

Here the total weight moved W is $80+90=170$ pounds. The force in the direction of the acceleration is 90 pounds; opposing reaction on the moving system are the gravity pull of 80 pounds and the inertia reaction. Therefore,

$$90 = 80 + \frac{170}{32} \times a,$$

from which, $\qquad a = 1.88 \text{ ft/sec}^2.$

Example 2.—A (Fig. 132) $=500$ pounds, and $P=30$ pounds. If there is no friction, find the acceleration and the time required for the system to move 6 ft.

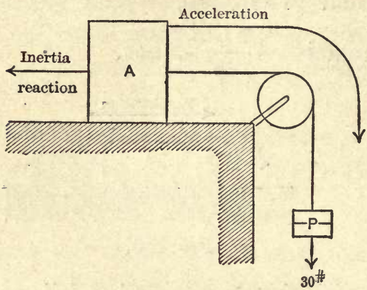


Fig. 132.

Here total weight W to be moved, $=500+30=530$ pounds. The gravity pull on A is borne by the surface and the gravity pull on P alone causes motion. Therefore,

$$30=\frac{530}{32}a,$$

$$a=1.81.$$

From laws of accelerated motion, when $S=6$ and $a=1.81$,

$$6=\frac{1.81}{2}\times t^2, \quad t^2=\frac{12}{1.81},$$

$$t=2.58 \text{ seconds.}$$

Example 3.—A car weighing 1800 pounds is drawn vertically up a mine shaft. In starting the car the acceleration is found to be 20 ft/sec^2. Find the tension in the cable during start.

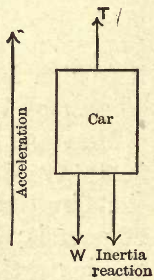


Fig. 133.

The "acceleration diagram" for the car is here as shown in Fig. 133.

The force causing motion is the pull T in the cable; the opposing actions are gravity pull, W, and inertia,

$$W = 1800 \text{ pounds};$$

$$\text{Inertia reaction} = \frac{1800}{32} \times 20.$$

(*a*) Therefore, $\quad T = 1800 + \frac{1800}{32} \times 20.$

From which, $\quad T = 2925$ pounds.

(*b*) Another solution may be given, thus:

$W = 1800$ pounds; and $a = 20$. Let T = pull in cable to give acceleration 20; then

$$T = \frac{1800}{32} \times 20,$$

$$T = 1125.$$

Add to this 1800 pounds, the car's weight, and so obtain the tension, 2925 pounds necessary to hold car up and also give the required upward acceleration.

(*c*) Still another solution is: Let $W = 1800$ and $a = 20 + 32 = 52$, that is, an acceleration *upwards* equal to the downward acceleration of gravity $+ 20$. Then

$$T = \frac{1800}{32} \times 52,$$

$$T = 2925.$$

Example 4.—W (Fig. 134) $=90$ lbs., $P=40$ lbs., friction between W and the plane $=3$ lbs., of pulley $=2$ lbs. Find the direction and the amount of the acceleration and the tension of cord both sides of pulley.

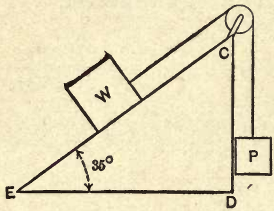


FIG. 134.

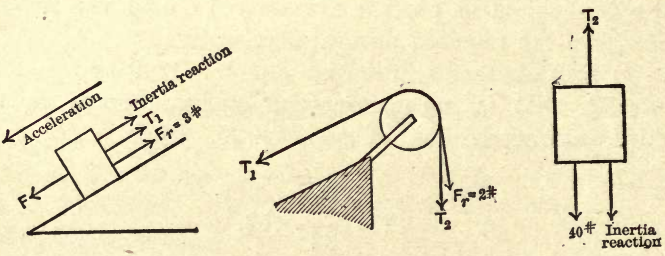
The force tending to move the system so that W will move *down* the plane is $90 \sin 35° = 51.7$ lbs. The forces opposing this motion are $P=40$ and friction $=5$ lbs. The force down the plane is greater and therefore change of motion will be down the plane. The total weight to be accelerated $=90+40=130$, and the inertia reaction $\frac{130}{32} \times a$.

Therefore, we may write the equation,

$$51.7 = 40 + 5 + \frac{130}{32} \times a.$$

From which, $\quad a = 1.65$ (downwards).

To find tension T_1 in cord between W and pulley, consider W alone. The acceleration diagram is as in Fig. 135(a).



(a) (b) (c)

FIG. 135.

Therefore, $$51.7 = T_1 + 3 + \frac{90}{32} \times 1.65.$$

or, $$T_1 = 44 \text{ lbs.}$$

Then, from pulley (Fig. 135(b)) we have, neglecting mass of the pulley,

$$T_1 = T_2 + 2,$$

$$T_2 = 42.1 \text{ lbs.}$$

Or, T_2 may be found from the equation for P alone (Fig. 135(c)),

$$T_2 = 40 + \frac{40}{32} \times 1.65,$$

$$T_2 = 42.1.$$

Example 5.—A weight $B = 1000$ lbs. is supported by a

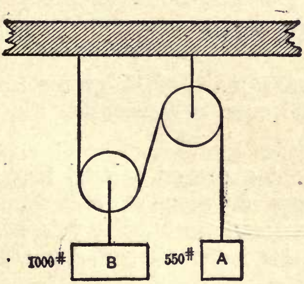


Fig. 136.

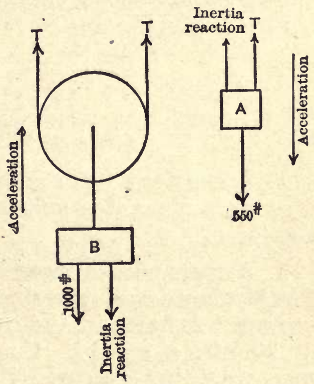


Fig. 137.

combination of pulleys as in Fig. 136. Load $A = 550$ lbs. With what acceleration will the combination move and

what will be tension in the rope? (Neglect friction and weight of pulleys.)

It will be seen from the figure that motion of A is twice that of B. Therefore acceleration of A will be twice the acceleration B.

Let a = acceleration of B. Then $2a$ = acceleration of A. Let T = tension of rope; since there is no friction the tension everywhere in the rope is the same. The diagrams for A and B are as in Fig. 137.

Therefore we may write the equations:

$$(1) \qquad T + \frac{550}{32} \times 2a = 550,$$

$$(2) \qquad 2T = 1000 + \frac{1000}{32} \times a.$$

These simultaneous equations may be solved for T and a in the usual way.

PROBLEMS

1. A constant force acting upon a mass of 15 grams for 4 seconds gives it a velocity of 20 cms. per second. Find the force.

2. A man pushes steadily against a car on a level track for 1 minute and thereby produces a velocity of 12 ft. a second. A horse in the same time can produce a velocity of 20.5 ft. a second. Compare the force exerted by the man with that exerted by the horse.

3. A force A can produce a velocity of 100 in 5 seconds, on a certain mass. A force B can produce on the mass a velocity of 50 in 25 seconds. How many times as large as B is the force A.

4. If gravity on the earth gives all bodies an acceleration of 981 cms/sec^2, and on the moon an acceleration of 163 5 cms/sec^2, compare the force of gravitation on the two spheres.

5. A mass of 2 lbs. at rest is struck and starts off with a velocity of 10 ft. a second. Assuming the time during which the blow lasts to be $\frac{1}{100}$ of a second, find the average force acting on the mass.

6. A car slides down a smooth incline with slope 1 ft. drop in 5 ft. length of track. Find (*a*) acceleration; (*b*) velocity acquired in 8 seconds; (*c*) distance traversed in 5 seconds.

7. Sand will stop a certain projectile fired into it in $\frac{1}{10}$ of a second. Loam will stop the same body going twice as fast in $\frac{1}{8}$ of a second. Compare the resistances to penetration offered by sand and loam.

8. What velocity would a train acquire in half a minute on a 3 per cent grade if there were no friction and what distance would it travel in that time?

9. A body weighing 90 lbs. is moving at the rate ot 80 ft/sec. What force expressed in pounds will stop if in 2 seconds? What force in $\frac{1}{10}$ of a second?

10. In an apparatus such as shown in Fig. 131, *A* and *B* weigh 10 lbs. and 9 lbs. respectively, and hang by a flexible string over the frictionless pulley *W*.

(*a*) Find acceleration of the system;
(*b*) Distance traversed in two seconds.

11. In an Atwood's machine the weights carried by the thread are $6\frac{1}{2}$ ounces each. The friction is equivalent to a weight of $\frac{1}{2}$ ounce. When the "rider," which weighs 1 ounce, is in position, what will be its gain in velocity per second? (See Fig. 127.)

12. A train weighing 1000 tons is going 30 miles an hour. The brakes can apply a force = 10 tons weight. If on a level, neglecting friction of track, etc., how long will it be before the train will stop? How far will it go before stopping?

13. Answer above if train is going up an incline of 1 in 20.

14. A body on a plane rising 1 in 10 goes 100 ft. in 10 seconds. Find force of friction.

15. In the apparatus shown in Fig. 132, $A = 100$ lbs., $P = 5$ lbs. No friction. How fast will A be moving after going 10 ft. along the table?

16. If $A = 900$ lbs. and friction amounts to force of 8 lbs., what must be the weight of P if A is to move $\frac{1}{2}$ ft. from rest in $\frac{1}{2}$ second? (See Fig. 132.)

17. A man weighing 175 lbs. stands on the floor of an elevator which is descending with uniform acceleration of 1 ft/sec^2. What will be the pressure of his feet on the floor? What will be the pressure when elevator is *ascending* with same acceleration?

18. A rope winding on a drum hauls a car vertically up the shaft of a mine with an acceleration of 2 ft/sec^2. If car weighs $1\frac{1}{2}$ tons find tension in rope.

19. A railroad train is moving on a level at the rate of 30 miles an hour. The train weighs 400 tons. If steam is shut off what force due to the brake is required to stop the train within 200 ft. (Neglect other frictions.)

20. In the apparatus of Fig. 134, $CD = 5$ ft., $CE = 50$ ft., $W = 9000$ lbs., $P = 40$ lbs. Find velocity in 4 seconds.

21. A bullet weighing 2 ounces going at rate of 1000 ft/sec penetrates 15 in. into an oak post. Find average resistance to penetration offered by post.

22. If, in diagram Fig. 131, B is 18 gm., what must be the mass A to give the system an acceleration of 40 cms/sec^2?

23. If in Fig. 131 A is 80 gm. and B is 60, find

(*a*) Acceleration;
(*b*) Tension of the string.

24. If in Fig. 131 A is 80 gm. and B 60 gm., and A is moving upward with velocity of 90 cms. per second, how far and how long will it move before coming to rest?

25. If in Fig. 131 $A=6$ and $B=5$ lbs., how long will it be before the velocity will be 300 ft/sec?

26. In an Atwood's machine the weights are 90 gms. and 80 gms. The system moves from rest for 4 seconds, when the string breaks. How long will the 80-gm. weight continue to rise, and how far will it rise? (See Fig. 127.)

27. A platform sustaining a weight of 120 lbs. is descending with a uniform acceleration of 4 ft/sec². Find the pressure of the weight on the platform.

28. Find the pressure between weight and platform in the case above if the platform is ascending.

(*a*) With uniform velocity of 9 ft/sec.

(*b*) With a uniform acceleration of 10 ft/sec².

29. In an inclined plane, slope $AC=20$ ft., height $CD=4$ ft. Body weighing 100 lbs. rests on the incline.

(*a*) Find acceleration along AC.

(*b*) Find pressure on AC.

(*c*) Find time to pass from C to A.

30. In the inclined plane of Fig. 134, $EC=13$ ft., $CD=5$ ft., $W=91$ lbs., $P=100$ lbs. Find

(*a*) Direction of motion and the acceleration.

(*b*) Tension in the string during motion.

31. In the incline plane of Fig. 134, $W=210$ lbs., $EC=28$ ft., $CD=6$ ft.; force of friction $=10$ lbs. Find mass of P,

(*a*) To produce equilibrium.

(*b*) To give acceleration of 8 ft/sec² *up* the plane.

32. A mass of 80 gms. slides down a smooth incline whose height is $\frac{1}{4}$ its length and draws another mass from rest, over a distance of 10 ft. in 3 seconds along a horizontal table level with the top of the plane, the string passing over the top of the plane. Find the mass on the table.

33. A body is projected up an incline rising 2 meters for each 49 meters of track with an initial velocity of 200 meters per second. Find the velocity and distance at the end of 6 seconds.

34. In what time would the body of Prob. 33 come to the highest point it will reach on the incline?

35. A car slides down an incline 80 meters long and having an inclination of 45 degrees to the horizontal. Find the velocity at the bottom and the time of the descent.

36. An elevator loaded with passengers weighs 1450 lbs. It is counterbalanced with weights weighing 950 lbs. With what acceleration would the elevator descend if it were free to fall and there were no friction on the pulleys or guides?

37. A weight of 2000 lbs. is supported by a system of pulleys, as indicated in Fig. 136. There is at A a weight of 1200 lbs. Find the time that it will take the weight A to fall 100 ft.

38. A N. Y., N. H. & H. electric locomotive, weighing 200 tons and hauling a 200-ton train, exerts an average tractive force of 10,000 lbs. Neglecting train friction, what time will be required to get up a full speed of 60 miles an hour from rest?

39. What time would be required by the train of Prob. 38 if on a 1 per cent up grade?

40. If the train of Prob. 38 starts up a 3 per cent grade at a speed of 55 miles per hour, what will be its speed at the end of a half mile?

41. If the locomotive of Prob. 38 is required to get up full speed in 200 seconds, what is the maximum trailing load it can haul?

87. Force and Acceleration in Rotary Motion.—In the preceding paragraphs of this chapter, we have shown that inertia is the property of all matter by reason of which it offers resistance to linear acceleration; that inertia depends solely upon mass; and that the fundamental relations in motion of translation are expressed by the equation, *force = mass × acceleration.*

When we come to apply these ideas to motion of rotation, we find that, because of the nature of such motion, certain other factors must be taken into account. Thus, the ability of a force to produce rotation does not depend upon its amount alone but also upon its moment arm. A small force having a long moment arm will produce as much tendency to rotation (i.e., torque) as a greater force having a correspondingly short arm. The resistance offered by a body to acceleration in rotation (i.e., *its rotational inertia*), does not depend upon mass alone but *also upon the way in which the mass is distributed with respect to the axis of rotation. The inertia reactions of masses farther out from the axis have greater moment arms and therefore react with greater effect in opposing rotation.* Thus it is a matter of common experience that a wheel with its mass mainly in its rim offers much greater resistance to starting and stopping than one which weighs exactly the same but which has its material concentrated near its axle. Then, also, in rotation, the *linear acceleration* given the particles of the body *must be greater in proportion to their distances out from the axis of rotation.*

If, however, we express the tendency to produce rotation in terms of *units of moment*, the acceleration as *angular acceleration*, which, as we have seen, is constant for all portions of the body, and if we assume that *a body offers unit resistance to angular acceleration* (i.e., has unit rotational inertia), *when one unit of moment* (unit torque) *is required to give the body unit angular acceleration,* we may write as an equation expressing the fundamental relations in rotary motion:

MOMENT OF FORCE (ALSO CALLED TORQUE) = ROTATIONAL INERTIA × ANGULAR ACCELERATION.

In this equation UNIT MOMENT OF FORCE = a force of 1 lb., having a moment arm of 1 ft., a force of 1 gm. having an

arm of 1 cm., a force of 1 dyne having an arm of 1 cm., etc., or their equivalents, according to the system of forces and lengths used. These may be called simply: *One pound-foot, one gram-centimeter, one dyne-centimeter,* etc.

UNIT ANGULAR ACCELERATION = 1 radian per sec^2.

Angular acceleration expressed as revolutions per $sec^2 \times 2\pi$ = acceleration expressed as radians per sec^2, since there are 2π radians in a complete circumference; therefore, the general equation for rotary motion may also be written:

$$\text{Torque} = \text{rotational inertia} \times 2\pi \times \text{acceleration in revolutions per sec}^2.$$

88. Moment of Inertia.—The resistance which a body offers to angular acceleration (i.e., its rotational inertia) is commonly called its *moment of inertia.*

UNIT MOMENT OF INERTIA* is the moment of inertia of a body to which *unit moment of force* will impart *unit angular acceleration.*

89. Radius of Gyration.—Attention has already been called to the obvious fact that in rotary motion portions

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* It may be shown that the moment of inertia of a particle is equal to the mass of the particle m, multiplied by the square of its distance r from the axis of rotation or mr^2. The moment of inertia I of a body equals the sum of the moments of inertia of its separate particles, or $I = \text{sum } (mr^2)$.

To compute the moment of inertia of a body, therefore, it is necessary to multiply the mass of each of its particles by the square of its distance from the axis and then to find the sum of all such products. This is impossible for irregular bodies of varying density, and in such cases I must be found by experiment. In case of regular homogeneous bodies the moments of inertia with respect to given axes may be calculated by the methods of the calculus. This is beyond the scope of this text and the reader who is interested is referred to more advanced treatises and to tables of moments of inertia, etc.

of the rotating body at different distances from the axis have different linear velocities. It is evident upon consideration, however, that there must be a point in every rotating body which will move with *such a velocity that if the entire mass of the body could be concentrated at this point, the resistance, which the body would offer to a torque tending to change its angular velocity, would be unchanged.*

The distance from the axis of rotation to this point is known as the *radius of gyration* for the body.

In rotary motion we may, therefore, when more convenient, consider the entire mass of the body as having the same linear velocity as a point situated a distance equal to the radius of gyration from the axis of rotation.

The value of the radius of gyration for various common forms as disks, cylinders, etc., is given in most practical handbooks. In the case of fly-wheels having a heavy rim it is customary to neglect the mass of the hub and spokes and regard the radius of gyration as the distance from the axis to middle of rim as r_0, Fig. 138.

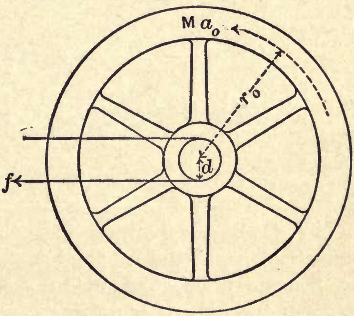

FIG. 138

90. Computation of Torque and Rate of Increase or Decrease of Speed for Fly-wheels, Shafts, etc.—If f is the force in *absolute units* which causes a fly-wheel to rotate and d the moment arm of this force with respect to the axis about which the wheel turns (see Fig. 138), then fd = torque which produces rotation.

If M = mass of rim and a_0 = *linear acceleration* at the

distance r_0 from the axis, then Ma_0 is a measure of the inertia reaction of the rim, and Ma_0r_0 = moment of this reaction with respect to the axis. Therefore

$$fd = Ma_0r_0.$$

This is the fundamental equation for the relation of torque, mass, and acceleration for rotating bodies where radius of gyration is known.

As, however, it is customary to express velocity of rotation in revolutions per second, and acceleration of the rotating body in terms of revolutions per second gained or lost each second, a more convenient form of the same equation may be derived as follows:

Let a_r = acceleration in revolutions per sec². Then acceleration a_0 in radians $= 2\pi a_r$, and linear acceleration = acceleration in radians × radius, or

$$a_0 = 2\pi r_0 a_r,$$

Substituting this value for a_0 in the equation above, we have

$$fd = Mr_0^2 2\pi a_r.$$

Note. Since moment of inertia (or rotational inertia) I = sum (mr^2), and the whole mass M of a body may be regarded as concentrated at a distance equal to the radius of gyration r_0 from the axis, it is evident that

$$I = Mr_0^2.$$

The equations,

Torque = moment of inertia × angular acceleration, or

$fd = I \times$ acceleration in radian/sec², and

$fd = Mr_0^2\, 2\pi a_r$,

are therefore identical, as $I = Mr_0^2$, and $\overline{2\pi a_r}$ = angular acceleration.

If force is to be expressed in gravitational units F, this expression becomes,

$$Fd = \frac{W}{g} r_0^2 2\pi a_r.$$

This is the equation commonly used for practical computations for fly-wheels, shafts, etc., where the radius of gyration, r_0, is known. Force F is here usually the difference in tension between tight and loose sides at the driving belt, and d is the radius of the belt pulley.

Example.—A fly-wheel whose rim weighs 8000 lbs. is started from rest by a pull of 1000 lbs. acting at the circumference of a belt pulley of 12 in. diameter. Outside diameter of fly-wheel is 6 ft. 6 in., thickness of rim 6 in. How long will it take to reach a speed of 120 revolutions per minute? (Neglect friction.)

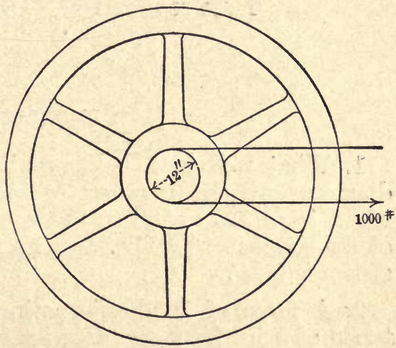

Fig. 139.

Fly-wheels are usually designed to have as large moment of inertia as possible for given conditions. The material is therefore mainly in the rim. Since the spokes and hub are light in comparison and the material near the axis offers relatively little resistance to angular acceleration, it is customary to neglect the mass of spokes and hub and to take as the *radius of gyration the distance from center of axis to middle of rim.*

Radius of gyration is here 3 ft. Mass of rim $\frac{8000}{32}$ units, moment arm of 1000 lbs. force $\frac{1}{2}$ ft. Therefore, from our expression $Fd = \frac{W}{g} r_0{}^2 2\pi a_r$,

$$1000 \times \tfrac{1}{2} = \frac{8000}{32} \times 3 \times 3 \times 2 \times 3.14 \times \text{accel. in revs. per sec}^2.$$

Or,

$$\text{Accel.} = .035 \text{ revolutions per sec}^2.$$

120 revs. per min. = 2 revs. per sec. to be gained.

Therefore,

$$\text{Time required} = \frac{2}{.035} = 57 \text{ sec. approx.}$$

PROBLEMS

1. What pull in a belt running on a belt pulley 20 in. diameter will be required to increase the speed of a fly-wheel from 120 r.p.m. to 180 r.p.m. in 10 seconds if weight of fly-wheel is 1600 lbs. and its radius from axis to center of rim is 2 ft.?

2. A force of 10 lbs. is acting on a fly-wheel at a distance of 3 in. from the axis for a time of 5 seconds. Find the speed and the total number of turns, starting from rest, made by the wheel if it weighs 64 lbs. and is of such a size and shape that $r_0 = 9$ in.

3. A uniform circular disk 4 ft. in diameter weighing 1000 lbs. is caused to rotate from rest by a force of 100 lbs. acting at its circumference. What will be the linear velocity of a point on the circumference at end of $1\frac{1}{2}$ minutes?

NOTE. For uniform circular disk, axis at center, $r_o =$ approximately $\frac{7}{10}$ of radius.

4. What force acting at the rim of the disk of Prob. 3 will be required to change its speed from 300 r.p.m. to 120 r.p.m. in 20 seconds?

91. Centrifugal Force.—Because of their inertia, bodies set in motion tend to continue moving in a straight line. Thus the body *m*, Fig. 140, moving from the positions M_1, M_2, etc., tends to go along the straight paths, M_1N_1, M_2N_2, etc. To pull it out of a straight path, and *compel it to move in a circle, requires a force acting constantly toward the center O.* This force toward the center, which is necessary to make a body rotate about this center, is called the *centripetal force.* The equal and opposite reaction to the centripetal force is called *centrifugal force.* It is this "centrifugal force" which sometimes causes rapidly-rotated fly-wheels, emery wheels, etc., to burst.

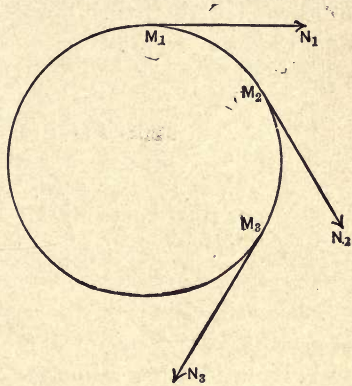

FIG. 140.

It should be noted that centrifugal force is *not a force in the usual sense of the word, but is a reaction due to inertia.*

It can be shown (see foot-note page 172) that the force that must act toward the center to keep a body moving at a uniform velocity in a circular path *depends upon the mass of the body, upon the square of the velocity, and upon the size of the circle;* and that the relationship between these quantities is expressed by the equation,

$$\text{Centripetal force} = \frac{mv^2}{r} \text{ (absolute units).}$$

In the gravitation system which we commonly use, this equation becomes

$$C. F. = \frac{WV^2}{gr},$$

where W is the weight of the body, V its velocity expressed in linear distance traveled per second, and r the radius of the circular path. Since "centrifugal force" is equal and opposite to centripetal force, we may also write the equation,

$$\text{Centrifugal force} = \frac{mv^2}{r} = \frac{WV^2}{gr}.$$

The rotating parts of all machinery must be carefully balanced about the axis, otherwise centrifugal force may cause injurious wear at bearings, jarring, etc.

Example.—What is the centrifugal force of each pound of weight of the rim of a fly-wheel 6 ft. diameter, making 400 revolutions per minute?

$$\text{Velocity} = \frac{6 \times \pi \times 400}{60} = 126 \text{ ft/sec approx.}$$

$$\text{Therefore centrifugal force} = \frac{1 \times 126 \times 126}{32.2 \times 3} = 164 \text{ lbs.}$$

NOTE.—Suppose a body of mass m to be moving with uniform velocity v around the circle ACE, Fig. 141. Let t be the time required to move over the arc AC. In the same time, the body has been pulled out of its straight path an amount equal to the distance BC by a force which, as the path is circular, must everywhere be directed toward the center O. If a be the acceleration which this force will impart to the mass m, $BC = \frac{at^2}{2}$. If the time t be sufficiently short, arc AC will be so short that it may be regarded as a straight

PROBLEMS

1. In a machine running at 1800 revolutions per minute there is an unbalanced weight of one pound at a radius of one foot. Find pull on the bearings due to centrifugal force.

2. A bicycle and rider weighing together 200 lbs. run around a curve of radius 50 ft. at a rate of 12 miles per hour. Find force with which tire must stick to track to prevent slipping.

3. How many revolutions per minute will suffice to break a string of tensile strength of 10 lbs. if a body weighing 5 lbs. be swung round at the end of 6 ft. of the string?

4. A train of 200 tons weight is rounding a curve of $\frac{1}{2}$ mile radius with a velocity of 30 miles per hour. What is the horizontal pressure on the rails?

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line: figures ACD and ACE will then be similar triangles. Therefore,

$$(1) \qquad \frac{AD}{AC}=\frac{AC}{AE}.$$

$$(2) \qquad (AC)^2=AD\times AE.$$

But $\quad AC=vt$ (distance moved in uniform motion);

$$AD=BC=\frac{at^2}{2} \text{ (distance moved, accelerated motion).}$$

And

$AE=2r$, r being the radius of the circle.

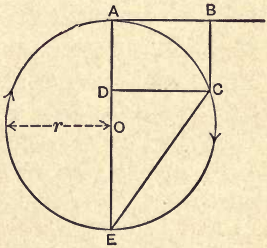

FIG. 141.

Substituting these values in equation (2) and solving, we have $a=\frac{v^2}{r}$, or the acceleration imparted by the force acting toward the center of the circle equals the square of the velocity divided by the radius of the path. And since force equals mass multiplied by acceleration, we have

$$\text{Centripetal force}=\frac{mv^2}{r}.$$

5. What should be the elevation of the outer rail for a standard gauge track of 4 ft. $10\frac{3}{4}$ in. for the train of Prob. 4?

6. Find the speed at which a simple Watt governor runs when the arm makes an angle of 30° with the vertical. Length of arm from center of pin to center of ball 18 in.

7. The rim of a cast-iron fly-wheel has a mean radius of 2 ft. Its section is 6 in. broad and 4 in. thick. It is revolving at 180 r.p.m. What is the centrifugal force per inch length of rim?

CHAPTER X

WORK, POWER, ENERGY

92. Work.—When a force causes a body to change its position against a resistance opposing such change, *work* is done on the body. Thus work is done upon a body when the body is lifted against the force of gravity; when the body is moved against the resistance offered by friction; or when the velocity of the body is increased or decreased against the reaction offered by inertia.

Two items must always be present when work is done; viz., *force* (equal to the opposing resistance), and *displacement.* The *measure of the work done is the product of the force and the displacement in the direction of the force.* Or,

$$Work = Fs.$$

Where the displacement is not in the line of action of the force, either the component of the displacement in the direction of the force must be found, or the component of the force in the direction of the displacement. The work is then *equal to the product of the force and the component of the displacement in the direction of the force;* or, it is *equal to the product of the displacement by the component of the force in the direction of the displacement.* Thus, suppose a body weighing 100 lbs. is to be pushed 10 ft. up a smooth plane rising 30° to the horizontal. The resistance here is due to the vertical gravity pull of 100 lbs., the displacement in the direction of this force is 10 sin 30° or 5 ft.

Hence the work done is $100 \times 5 = 500$ ft.-lbs. Or, the component of the force in the direction of displacement is $100 \sin 30° = 50$ lbs., and the work done is therefore $50 \times 10 = 500$ ft.-lbs. Or, suppose a body weighing 200 lbs. resting upon a horizontal table is pushed 3 ft. along the table by a force of 60 lbs. acting at an angle of 20° with the horizontal. The force in the direction of displacement is here $60 \times \cos 20° = 56.4$ lbs. The work is therefore $56.4 \times 3 = 169.2$ ft.-lbs. The weight of the body and the component of the force perpendicular to the table do no work as they produce no displacement in their directions.

93. Units of Work.—Quantity of work done is measured by the product force $\times$ distance. Work may therefore be expressed in different units, dependent upon the units used in defining the force and the displacement. Thus if force is measured in *pounds* and displacement in *inches*, the work will be expressed in *inch-pounds*. Other units are centimeter-grams, foot-tons, etc.

The units of work commonly used are the *foot-pound* and the *erg*.

A FOOT-POUND *is the work done when a force of one pound acts through a distance of one foot in the direction of the force;* or when *an equivalent amount of work* is done, as when a force of 2 lbs. acts through a distance of $\frac{1}{2}$ ft., a quarter pound acts through a distance of 4 ft., etc.

AN ERG *is the work done by a force of one dyne acting through a distance of one centimeter.* The erg is thus a very small amount of work; for convenience of expression, therefore, the JOULE $= 10^7$ ergs is often used.

94. Work Diagrams.—Since work equals the product of force and distance, it may often be conveniently represented by the area of a diagram. Thus, suppose a constant force of 100 lbs. acts through a distance of 16 in.; we may represent the force by a vertical line 5 units long

and the distance by a horizontal line 4 units long, then the *area* of the figure *represents the work in terms of the scale used.* A unit on the vertical scale here represents 20 lbs., one unit on the horizontal scale represents 4 in. of distance. *One square unit, therefore, represents $20 \times 4 = 80$ inch-pounds* of work, and the whole area $5 \times 4 = 20$ sq. units represents $20 \times 80 = 1600$ in.-lbs.

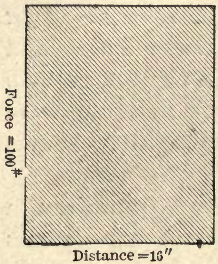

FIG. 142.

If the force doing work is not constant, the work done may still be represented by an area. Thus, suppose Fig. 143 represents the conditions in a steam-engine cylinder where, as is usually the case, the steam pressure is not constant during the entire stroke. The vertical height of the diagram at any point represents the steam pressure in pounds per square inch at that point in the stroke. The area of the figure (=*average height*×*length*) therefore represents, in terms of the scale to which it is drawn the *product of average steam pressure times length of stroke or the work done by the steam on each square inch of the piston as it moves forward the length of one stroke.*

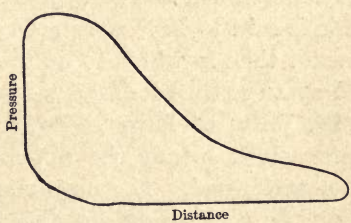

FIG. 143.

Suppose the area of the diagram is 2.2 sq. in. and suppose $\frac{1}{10}$ in. = 1 in. is the horizontal scale, and 1 in. = 30 lbs. per square inch is the vertical scale. Then each square inch of the diagram represents $10 \times 30 = 300$ in.-lbs., and the whole

diagram represents $2.2 \times 300 = 660$ inch-pounds or 55 foot-pounds.

The area of irregular figures such as the above may be found approximately by measuring the height at 10 or more points at equal intervals along its entire length and taking the average of these measured values as the average height. Average height×length=area. A more accurate method is to use an instrument for measuring areas known as a planimeter.

Work diagrams similar to Fig. 143, taken from engines in actual practice, are known as *indicator cards;* instruments for taking indicator cards are known as *steam-engine indicators.* In practice, steam-engine indicators are "calibrated" so that the values of the vertical and horizontal scales of the diagram in terms of actual steam pressure and length of stroke are known.

95. Power.—In the previous discussion of work it will be noticed that the *time required* for the force to act through a given distance *was not considered;* the total work being the same whether performed quickly or slowly. But a machine which will do a given amount of work in a short time is said to be "*more powerful*" than one that requires greater time in which to do the same work.

POWER is therefore defined as *the rate of doing work.*

96. Units of Power.—Power may be expressed in any units which specify the work done and the time in which it is performed. The customary units of power are, however, the *horse-power* and the *watt.*

Work is done at the rate of ONE HORSE-POWER (abbreviated H.P.) *when 33,000 foot-pounds of work are done in one minute.* Therefore,

$$\text{H.P.} = \frac{\text{foot-pounds in one minute}}{33{,}000}.$$

Work is done at the rate of ONE WATT when *one joule of work is done per second.*

$$1 \text{ H.P.} = 746 \text{ watts.}$$

$$1 \text{ kilowatt} = 1000 \text{ watts} = \frac{1000}{746}, \text{ or about } 1\tfrac{1}{3} \text{ H.P.}$$

In measuring electrical power the product of the electrical pressure by the current in amperes which it is producing = watts. Or watts = volts $\times$ amperes.

PROBLEMS

1. A car weighing 3000 lbs. and containing 2 tons of coal is hoisted up the shaft of a mine 400 ft. deep. Work done?

2. A magnet is moved 124 cm. against a resistance of 1200 dynes. Work done?

3. Weight of a trolley car is 8 tons. Average resistance to motion due to friction, etc., is one-sixteenth of the weight. Work done in run of 4 miles on a level track? Power required if run is made in 20 minutes?

4. James Watt found that an ordinary English dray-horse could travel an average of $2\frac{1}{2}$ miles per hour and raise 150 lbs. by means of a rope and pulley. Find the work done per minute.

5. A man weighing 180 lbs. walks up stairs, a height of 20 ft. in $\frac{1}{2}$ minute. At what rate does he work?

6. A load is hauled along a level floor by means of a rope inclined 30° to the horizontal. If pull in rope is 100 lbs. and the body is moved 10 ft., compute work done.

7. Find H.P. of the engine that should be used for raising coal from a pit 200 ft. deep, working 15 hours a day, if the average daily yield is 1000 long tons.

8. A train weighs 220 tons. The resistance to motion is 20 lbs. per ton on a level. Find H.P. of an engine which can just keep it going 30 miles an hour up a grade of 10 ft. rise per mile of track.

9. A 5-H.P. engine is used to pump water from a shaft 100 ft. deep. How many cubic feet will it raise in 24 hours? (Cubic foot of water weighs 62.5 lbs.)

10. What H.P. is required to supply 1000 families with 60 gals. water each per day of 10 hours, if water is pumped to a height of 200 ft.? (Gallon of water weighs 8 lbs.)

11. Find the work done against a brake which gives a frictional resistance of 15 lbs. on the rim of a wheel of 2 ft. radius, while the wheel makes 50 revolutions.

12. Find the H.P. used in the preceding problem if the 50 revolutions required 20 seconds time.

13. Compute the H.P. developed by an engine under the following conditions: Diameter of piston 4 in.; length of stroke 5 in.; mean effective pressure 42 lbs. per square inch; number of revolutions 275 per minute.

14. A body 4 ft. $\times$ 3 ft. $\times$ 2 ft. rests upon 2×3 face. Body weighs 150 lbs. Compute the work done in turning it over on 2 ft. edge.

15. An engine hauls a car 10,000 ft. along the track. The pull exerted by the engine upon the car in the beginning was 3500 lbs. and as recorded every 1000 ft. was as follows: 3500, 3250, 2800, 3450, 3700, 3600, 3550, 3300, 2950, and 2750 lbs. Construct a "work diagram" for above data. Determine area of diagram and from this the total work done on car.

16. A punch exerts an average pressure of 50 tons in punching a hole through a plate $1\frac{1}{8}$ in. thick. Compute work done.

17. A well 6 ft. diameter and 30 ft. deep is dug. Compute work done in raising the material if 1 cu. yd. weighs 4000 lbs.

18. The weight of a train is 240 tons and the draw-bar pull is 8 lbs. per ton. Find H.P. required to keep train running 20 miles per hour.

19. What is the difference in tension between the two sides of a belt running 3000 ft. a minute and transmitting 250 H.P.?

20. Find the speed of a driving pulley $2\frac{1}{4}$ ft. diameter to transmit 10 H.P., the driving force of the belt being 120 lbs.

21. An automobile that weighs 2 tons goes up a rough road of 1 ft. rise in 10 ft. of road at a speed of 15 miles per hour. Frictional resistance is 18 lbs. per ton. Compute H.P. developed.

22. Average width of an indicator diagram for one end of a piston is 1.6 in. For the other 1.5 in., and 1 in. represents 36 lbs. per square inch. Piston is 10 in. diameter, stroke 1 ft., revolutions per minute 120. What is the indicated H.P. if both sides of the piston are equal?

23. What is the H.P. of a stream that has a section of 10 sq. ft., velocity of 3 miles per hour, and a 12-ft. fall?

24. A fire-pump will deliver 1000 gals. of water per minute at 100 lbs. pressure. Compute work expended in pumping. (1 lb. pressure = 2.34 ft.-head.)

97. Energy.—A body which is capable of doing work is said to possess energy.

Thus, we say that a clock spring when coiled possesses energy because, in unwinding, it can do work in driving the clock mechanism, that a moving projectile possesses energy because it can overcome the resistance offered by the air, by armor plate, etc., and thus do work. A storage-battery possesses energy because it can furnish an electric current to operate a motor, etc. Hence,

Definition: ENERGY IS THE CAPACITY FOR DOING WORK.

98. Source of the Energy of a Body.—A body or system of bodies possess energy as a result of *the fact that at some previous time work has been done upon it.* Thus in the illustrations just given, the coiled spring received its energy when work was done on it in winding it up; the moving

projectile acquired the energy it did not have while lying at rest in the cannon when the gases produced in the explosion of the powder, by their presure, did work upon the projectile, thus overcoming the resistance offered by its inertia and imparting motion; the storage-battery received its energy when "charged" from the dynamo machine. Work is therefore *the act of transferring energy. The body which is doing the work gives or expends energy, the body worked upon receives energy.*

The energy thus transferred may be transformed into heat during the process, and thus be no longer available for doing work; or a part of it may be stored up as available energy in the body worked upon, as when this body is raised to a higher level, when it is put in motion, etc. When energy is thus stored, the body worked upon becomes in turn capable of doing work.

The amount of available energy possessed by a body is measured by the amount of work the body can do and is therefore expressed in the same units as work, i.e., in foot-pounds, ergs, etc.

99. Kinetic and Potential Energy.—Mechanical energy is of two kinds:

First, *Energy of Position*, or *Potential Energy*. Thus, if a compressed spring can in extending exert a force of 60 lbs. over a distance of 4 in., it has when compressed a *potential energy* of 240 in.-lbs.; or, if a body weighing 10 lbs. is suspended so that when released it may freely fall under the action of gravity 6 ft., it has a *potential energy* due to its position of 60 ft.-lbs.

Second, *Energy due to Motion, called Kinetic Energy.* A body in motion can be brought to rest only through a resistance opposing the motion. The product of this resistance by the distance through which it acts in stopping the body measures the work stored in the body, and there-

fore the kinetic energy which it possesses. Thus the kinetic energy of a moving train = the work done by it while stopping = the resistance due to friction, brakes, etc., $\times$ distance moved over while coming to rest.

The kinetic energy of a body may be computed from its mass and velocity as follows:

Kinetic energy = work a body can do because of its motion = retarding force $\times$ distance through which it acts in stopping body $= F \times s$. But

$$F = ma.$$

Therefore, $$\text{K. E.} = Fs = mas.$$

When a body is accelerated or retarded, $a = \frac{v}{t}$ and $s = \frac{at^2}{2}$, therefore, by substituting these values in the equation above, we have,

$$\text{K. E.} = \frac{mv^2}{2}.$$

In gravitational units, this expression becomes: $\text{K.E.} = \frac{Wv^2}{2g}$. In applying this equation, v and g must always be expressed in the same units.

100. Transformation of Energy.—*Potential energy* may be transformed into an equal amount of *kinetic energy*, and *vice versa*, the sum of the two kinds of energy always remaining constant unless some of it is used up in overcoming friction, or in doing other work. Thus, if a body has *potential energy*, due to its position, and is allowed to fall, its potential energy decreases as it descends. But its *kinetic energy* due to its velocity, increases at the same rate, until the body has, just as it is reaching the end of its fall, no potential energy, but an amount of kinetic energy equal to the potential energy which it had at the

start. If a body weighing W lbs. is suspended h ft. above the ground its potential energy is Wh ft.-lbs. If it falls freely its kinetic energy on striking the ground $=\frac{Wv^2}{2g}=Wh.$

Similarly, if a body has a certain amount of *kinetic energy*, due to its velocity, and if this velocity is directed upward, the velocity of the body, and therefore its kinetic energy, will *decrease* as it rises; but its *potential energy* will *increase* as it rises in the same ratio until, when its velocity has been reduced to zero and it is just ready to commence to fall again, its potential energy will exactly equal the kinetic energy which the body had in the beginning.

In addition to what has here been termed "mechanical energy," energy may exist in other forms, as heat, electrical energy, chemical energy, etc. Under proper conditions, energy of one form may be transformed into another form. Thus mechanical energy may be transformed into the form of molecular kinetic energy called heat, as, for example, in machinery where energy expended against friction is transformed into heat thus heating the bearings. In improperly constructed or poorly lubricated bearings the amount of energy thus transformed, i.e., the "wasted work," may be considerable, and thus the parts of the machine may be injured from undue heating.

Or, heat energy may be transformed into mechanical energy, as in the steam-engine.

Whenever heat energy is transformed into mechanical energy, or mechanical energy into heat, it has been definitely determined that the transformation always occurs in accordance with the law that:

The heat energy required to heat 1 lb. of water 1° F. = 778 ft.-lbs. (See also Art. 112.)

Mechanical energy when used for driving a dynamo may be transformed into the energy of an electric current, and

by means of a motor, energy of an electric current is transformed into mechanical energy. In electric lights, heaters, etc., electrical energy is transformed into heat.

PROBLEMS

1. A reservoir contains 50,000 gals. water at an average elevation 200 ft. above a given level. Potential energy of water with respect to this level?

2. What is the potential energy of a mass of 1500 lbs. at a height of 80 ft. above the ground? What will be its velocity on reaching the ground if allowed to fall freely and what will be its kinetic energy on striking?

3. A bullet weighing 15 gms. has a muzzle velocity of 600 meters per second. How far would the energy of the bullet raise a mass of 1 kgm. if it could all be used for that purpose?

4. A 5-ton pile-driver falls freely 6 ft. Assuming all of its kinetic energy to be expended in sinking the pile, against what average resistance can it drive the pile 1 in.?

5. If the mass of a moving body be doubled and its velocity at the same time be increased fourfold, how is the kinetic energy changed?

6. 6480 ergs of energy are expended by the action of a constant force urging a mass of 90 gms. a distance of 600 cm. along a smooth horizontal plane. Find

(*a*) The final velocity;
(*b*) The acceleration;
(*c*) The time;
(*d*) The force.

7. A car weighing 10 tons is moving at the rate of 36 miles an hour. Find average force required to stop it within a space of 50 ft.

8. A bicyclist weighing 155 lbs. mounted on a wheel weighing 25 lbs. rides at a velocity of 25 miles per hour on a horizontal plane. If he comes to a slope rising 12°

and removes his feet from pedals, how far up the slope will he go?

9. The steam hammer shown in the diagram (Fig. 144) has a cylinder 72 sq.in. in cross-section, the stroke is 20 in., the steam pressure is 80 lbs. per square inch, the weight of hammer, including piston and piston-rod, is 644 lbs. Find the energy which hammer will have after one full stroke. How far will the hammer blow sink into a piece of copper if the average resistance offered by the copper is 50 tons?

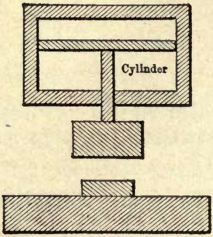

Fig. 144.

Note. Work done by gravity and work done by steam, *both* give velocity to hammer. Disregard friction.

10. The section of a stream is 10 square ft. and the average velocity of the water 2 ft/sec; there is an available fall of 30 ft. What is the available supply of energy per minute?

11. A ball weighing $\frac{1}{4}$ lb. strikes a shield with a velocity of 1000 ft/sec. It pierces the shield and moves on with a velocity of 400 ft/sec. Energy lost in piercing shield? Resistance offered by shield if it is 1 in. thick?

12. A fire engine pump is provided with a nozzle 1 sq.in. area of section and water is projected through the nozzle with an average velocity of 140 ft/sec. Find:

1. Kinetic energy of each pound of water as it leaves nozzle.
2. Horse-power engine required to drive the pump which supplies the nozzle, assuming that 75 per cent of the energy supplied to the pump is transferred to the water.

13. A rapid-fire gun discharges 5 projectiles per minute each of a weight of 100 lbs. and a velocity of 2000 ft/sec. What is the H.P. expended?

14. A hammer weighing $1\frac{1}{2}$ lbs. has a velocity of 18 ft/sec. when it strikes a nail. What is the force exerted on the nail if it is driven into the wood $\frac{1}{2}$ inch?

101. Energy of Rotating Bodies.—As shown in Art. 99, the kinetic energy of a body moving in a straight line is expressed by half the product of its mass by the square of its velocity. In such motion all portions of the body move in the same direction and with the same velocity. In the case of a rotating body, as for example, a fly-wheel, the kinetic energy of any *particular particle* of the wheel is obviously $\frac{1}{2}mv^2$, where m is the mass of the particle, v its velocity, and the kinetic energy of the whole wheel is the sum of the kinetic energy of its particles, or sum $(\frac{1}{2}mv^2)$, where v has different values according to the distance of the particles from the axis of rotation.

Since in rotary motion the whole mass of the moving body may be assumed as concentrated at a distance equal to the radius of gyration from the axis (see Art. 89), a convenient equivalent expression for the kinetic energy of a rotating body is

$$\text{K. E.} = \tfrac{1}{2}mv_0^2 = \frac{Wv_0^2}{2g},$$

where v_0 = the linear velocity at the distance of the radius of gyration from the axis.*

Example 1.—What is the kinetic energy of a fly-wheel weighing 3000 lbs. when running 180 revolutions per minute, its dimensions being as follows:

Outside diameter 6 ft. 6 in.;
Thickness of rim 6 in.;
Spokes and hub may be neglected.

---

* Since moment of inertia bears the same relation to rotary motion as mass does to linear motion, the kinetic energy of a rotating body may also be expressed as one-half the product of its moment of inertia by the square of its angular velocity, or,

$$\text{K.E.} = \tfrac{1}{2}Iv_a^2.$$

The radius of gyration for fly-wheels where spokes and hub may be neglected is approximately equal to the mean radius of the rim. Hence the kinetic energy in this case will be approximately as if all mass were placed 3 ft. from the center. Therefore,

$$v=\frac{6\times\pi\times 180}{60}=56.5\text{ ft/sec.};$$

$$\text{K. E.}=\frac{Wv_0^2}{2g}=\frac{3000\times 56.5\times 56.5}{2\times 32.2}=149{,}000\text{ ft.-lbs.}$$

Example 2.—A fly-wheel weighing 5400 lbs., mean radius 2 ft. 9 in. running at 120 turns per minute, gives up 8000 ft.-lbs. of energy; how much is its velocity diminished?

Let v_1 be velocity in beginning, and
v_2 be velocity after the 8000 ft.-lbs. have been withdrawn.

$$v_1=\frac{2\times 2\tfrac{3}{4}\times\pi\times 120}{60}=34.6\text{ ft/sec};$$

$$\text{K. E. in beginning}=\frac{5400\times v_1^2}{2\times 32.2};$$

$$\text{K. E. at end}=\frac{5400\times v_2^2}{2\times 32.2}.$$

Therefore, $$\frac{5400\times v_1^2}{2\times 32.2}-\frac{5400\times v_2^2}{2\times 32.2}=8000.$$

Substituting the value of v_1 above and solving,

$$v_2=32.2\text{ ft/sec}=115+\text{revolutions per minute.}$$

102. Experimental Study of Kinetic Energy of a Fly-wheel.—The student apparatus shown in Fig. 116 may be used to illustrate experimentally, the law of conservation of energy, the stored kinetic energy of a rotating body, and such terms as radius of gyration, moment of inertia, etc., which are apt to be hazy and very indefinite conceptions. For this purpose the cord leading from the scale pan ends in a loop passed over a pin in the drum and the length of cord is so fixed that the loop is released from the pin at the instant the weight reaches the floor. The energy supplied to the system while the weight descends is represented by the product Wh, where W is weight of scale pan, h the vertical distance through which it moves before it is released. This energy must have been expended in work against friction while the torque was being applied or be stored as kinetic energy in the moving pulley and scale pan at the instant the latter is released. By the law of conservation of energy, the total energy received must equal the energy expended + energy thus stored, or

$$Wh = \text{energy spent against friction} + \text{kinetic energy of weight} + \text{kinetic energy of pulley.}$$

To check this equation experimentally, the distance h may be measured, and the friction of the pulley determined in terms of the pull tangent to the drum required to keep the pulley turning at uniform speed. In an actual test, distance h was found to be 26.1 in., friction to be 10 ounces.*

Several determinations were then made with a stop-watch of the time required for the pulley to come to rest after

* Since the friction of the apparatus as used in the experiments on acceleration was too small to admit of satisfactory measurement, the cone bearings were tightened slightly for this test. They were then kept flooded with oil.

the torque had been removed, and of the number of revolutions made in the meantime. These agreed unexpectedly well as shown by the following table:

Observation.	Time in Seconds.	Revolutions.
1	21.0	42
2	20.0	39
3	20.0	39
4	20.0	40
5	21.0	42
6	20.4	41
7	21.0	42
8	20.0	40

The average time required to come to rest was 20.4 seconds, and the average number of revolutions 41. (In this as in subsequent computation, two significant figures only are kept, as the method did not seem to warrant greater claim to precision.)

The average velocity of the pulley while coming to rest was therefore $\frac{41}{20.4}=2.0$ revolutions per second. And since the final velocity was zero, at the time the weight was released the wheel must have been making 2.0×2 or 4.0 revolutions per second. The diameter of the drum on which the cord is wound is 3 in. Therefore the velocity of the weight when it struck the floor was $\frac{3}{12}\times\frac{22}{7}\times4.0=3.1$ ft/sec.

The scale pan weighed 10.3 lbs. Hence, substituting preceding values, we have

$$(a)\ \text{Energy supplied}=Wh=10.3\times\frac{26.1}{12}=22.4\ \text{ft.-lbs.}$$

(*b*) Energy expended in doing work against friction = force of friction × distance through which it was overcome = $\frac{10}{16}\times\frac{26.1}{12}=1.4$ ft.-lbs.

(*c*) Kinetic energy of weight $=\frac{Wv^2}{2g}=\frac{10.3\times3.1\times3.1}{2\times32}=1.5$ ft.-lbs.

(*d*) Stored energy of pulley = work done against friction in coming to rest = friction measured at the drum × circumference of drum × revolutions in coming to rest $=\frac{10}{16}\times\frac{3}{12}\times\frac{22}{7}\times4.1=20$ ft.-lbs.

Therefore, substituting these values in our expression of the law of conservation of energy—(*a*) should equal (*b*) + (*c*) + (*d*),—we have, 22.4 ft.-lbs should = 1.4 + 1.5 + 20 ft.-lbs., or 22.4 should equal 22.9. This result, while far from precise, illustrates the law very satisfactorily.

The kinetic energy of the pulley may be expressed by $\frac{w_ev^2}{2\times32}$, where v is taken as *rim speed*, and w_e is an expression for the "equivalent weight" of the pulley, i.e., the weight which concentrated at the rim and moving with rim speed would oppose the same rotational inertia to increase or decrease of speed, and would possess the same relation to energy changes as the true weight of the pulley, which is of course distributed in the hub, spokes, rim, drum, etc., with different actual linear speeds at each point. Equating this with the value for the kinetic energy as found in (*d*), and putting for v the computed rim speed of a wheel $13\frac{3}{4}$ in. diameter, making 4 revolutions per second, we have

$$\frac{w\left(\frac{13\frac{3}{4}}{12}\times\frac{22}{7}\times4\right)^2}{2\times32}=20\text{, from which } w_e=6.2 \text{ lbs.}$$

This differs of course from the actual weight of the pulley which is 13 lbs.

Or if, instead of assuming the rim speed as the velocity of the whole pulley and computing its *"equivalent weight"* at this speed as in the preceding, we assume the actual weight 13 lbs., and figure the speed with which this whole weight must move in order to possess the same kinetic energy as the actual pulley, we have the equation,

$$\frac{13v^2}{2\times32}=20, \text{ from which } v=9.9 \text{ ft/sec.}$$

This is the speed of a point 4.7 in. out from the axis of the pulley, or, in other words, what is commonly called the *radius of gyration* for the pulley is 4.7 in.

And since " moment of inertia," I, of pulley is equal to its mass times the radius of gyration squared,

$$I=13\times\frac{4.7}{12}\times\frac{4.7}{12}=2.0.$$

This agrees reasonably well with the value 1.9 which was obtained independently by a second experiment, using the torsional pendulum method.

Of course the above values for " equivalent weight," radius of gyration, and moment of inertia are approximations to the same degree that 20 ft.-lbs. expresses the kinetic energy of the pulley. They cannot be computed directly, however, because of the different materials in the various parts of the pulley, its irregularities in shape, the balancing weights attached to the rim, etc., and probably these values compare well with any which could be obtained experimentally.

PROBLEMS

1. The fly-wheel of a gas engine has following dimensions:

Mean diameter of fly-wheel, 6 ft.;
Width of rim, 8 in.;
Thickness of rim, 6 in.;
Weight of rim per cubic foot, 465 lbs.

What is the kinetic energy of the wheel when running at 150 revolutions per minute?

2. How much energy must be supplied to the wheel in Problem 1 to increase its speed to 180 revolutions per minute?

3. The rim of a fly-wheel weighs 15,000 lbs. and its mean linear velocity is 40 ft/sec. How many foot-pounds of work are stored in it? If it is required to give out 20,000 ft.-lbs., how much will its velocity be decreased?

4. A fly-wheel has a mean radius of 3 ft. 4 in., and a normal speed of 122 revolutions per minute. It is required to supply 3600 ft.-lbs. from its store of energy while slowing down to 120 revolutions per minute. What mass of rim is required?

5. A fly-wheel weighing 160 lbs. possesses 800 ft.-lbs. of kinetic energy. What is the velocity of the point where all its mass may be regarded as collected?

103. Momentum.—The product of the mass of a moving body by its velocity is called its *Momentum*. Or,

$$Momentum = mv.$$

Thus a body of 10 units mass when it is moving with velocity 8 units per second, has a momentum 80; a body whose mass is 20 units has also a momentum of 80 if its velocity is 4 units per second.

The starting and stopping, or *any* change in the velocity of a moving body is governed by the relation $f = ma$. If

v=total change in velocity, $a=\frac{v}{t}$. Hence the above equation may be expressed in the form $f=\frac{mv}{t}$, i.e., *the force acting may be measured by the momentum it can generate or destroy in unit time.*

Two methods may therefore be used in computing the force acting in producing a change of velocity. Thus, suppose we are given the *mass* and *velocity* of the body and *the distance* through which the force acts. We may then compute the kinetic energy of the body and apply the principle of work. For example, suppose a hammer weighing 5 lbs., moving with a velocity of 40 ft/sec when it strikes a nail, drives the nail 1 in. into the wood. The kinetic energy of the hammer is

$$\frac{5\times 40\times 40}{2\times 32.2}=124 \text{ ft.-lbs.}$$

This energy is expended in doing work against the resistance R offered by the wood, or $124=R\times\frac{1}{12}$, hence,

$$R=1490 \text{ lbs.}$$

It should be noted that R is here the *average resistance* offered by the wood, over the distance of 1 in. R is the *space average* of the force. The resistance offered by the wood may or may not be uniform; it is equivalent to the uniform resistance of 1490 lbs. through the whole distance, 1 inch.

Or, if we know the *mass* and *velocity* of the body and the *time* during which the force acts, we may compute the momentum of the body and then find the force from

the relation $f=\frac{mv}{t}$. Thus, suppose the hammer of the preceding illustration is brought to rest in $\frac{1}{200}$th of a second.

Then momentum of hammer $=\frac{5}{32.2}\times 40$. And the resistance R offered by the wood $=\frac{Wv}{gt}=\frac{5\times 40}{32.2\times\frac{1}{200}}=1240$ lbs.

R is here the *time-average* force, i.e., the average force which will stop the hammer in the given time, $\frac{1}{200}$ second.

This method is very useful when considering *impulsive forces or blows*, i.e., forces which act for a very short interval of time. The following examples will illustrate the general applications:

Example 1.—A stream of water 2 in. diameter impinges at an angle of 90° upon a brick wall. If the velocity of the stream is 200 ft/sec, what is the force exerted on the wall?

The weight of water reaching the wall *per second* is represented by a cylinder of water 2 in. diameter and 200 ft. long $=\frac{1}{12}\times\frac{1}{12}\times\frac{22}{7}\times 200\times 62.5=273$ lbs. The change of velocity is equal and *opposite* to the velocity of the stream, or 200 ft/sec. Therefore reaction of wall

$$=\frac{273\times 200}{32.2\times 1}=1700 \text{ lbs.}$$

Example 2.—120 cu.ft. of water leave the rim of the wheel of a centrifugal pump every minute. The component velocity of the water in the direction of motion of the rim is 20 ft/sec., and the velocity of the rim is 25 ft/sec. If the water enters at the center with no velocity in the

direction of the rim, what H.P. will be required to drive the pump, neglecting friction?

120 cu. ft. per minute = 2 cu. ft. each second = $2 \times 62.5 =$ 125 lbs. of water each second. To impart to this a velocity of 20 ft/sec requires a force of $\frac{125 \times 20}{32.2 \times 1} = 77.6$ lbs.

If the velocity of the rim is 25 ft/sec, the work required is $77.6 \times 25 = 1940$ ft.-lbs. per second. The H.P. required is therefore

$$\frac{1940 \times 60}{33{,}000} = 3.53 \text{ H.P.}$$

PROBLEMS

1. A truck weighing 12 tons moving with a velocity of 5 ft/sec is stopped by buffers in $\frac{1}{3}$ second. What is average force of the blow?

2. A ship of 10,000 tons moving at 5 miles per hour is stopped in one minute. Average force required?

3. A gun delivers 100 bullets per minute each weighing one ounce, with a horizontal velocity of 1610 ft/sec. What is the average force exerted upon the gun?

4. A $1\frac{1}{2}$ in. stream from a fire-engine is thrown horizontally against a wall with a velocity of 150 ft/sec. What force is exerted against the wall?

5. A centrifugal pump discharges 80 cu. ft. of water per minute at a velocity tangent to the rim of the wheel of 25 ft/sec. If the velocity of the rim is 30 ft/sec, what is the H.P. required?

104. Principle of the Conservation of Momentum. Since a force is measured by the momentum it will generate in unit time, it follows that *equal forces, acting for the same time, will generate equal momenta.* The momentum

imparted to a gun when fired, if the gun can be considered as free from other bodies, will equal the momentum imparted in the opposite direction to the bullet, although because of its greater mass the velocity of the gun will be very small in comparison; and in general, the same is true for any system of two bodies acted upon simultaneously by equal and opposite forces. When, for example, a body A strikes another body B, the mutual pressures between the bodies are equal and opposite during the time of impact. If A loses momentum, B must gain an equal amount provided the action of outside bodies upon both can be neglected. The total momentum of the system (i.e., of A and B taken together) therefore remains the same as it was before impact, although the velocities of both A and B may be changed in either amount or direction or in both. This fact that *the total momentum of a system remains constant unless a force external to the system is applied* is known as *The Law of Conservation of Momentum.*

It must not be inferred from what has been said, however, that the total *mechanical energy* of the system remains constant also. During impact, work is done against internal forces (cohesion, internal friction, etc.), and some mechanical energy is thus transformed into heat energy. The *mechanical energy is therefore always less after impact.*

CHAPTER XI

FRICTION. THE GENERAL LAWS OF MACHINES

105. Friction.—The resistance offered to the sliding of one body on another is called the force of friction between the two bodies.

Thus, suppose a cast-iron block weighing W pounds rests upon the horizontal surface AB, also of cast iron, Fig. 145. The pressure between the surfaces in contact will be W pounds. A force S_1 will be required to cause the body W *to start from rest; a smaller force,* S_2, *will keep the body moving uniformly when once started.* S_1 therefore measures *the force of starting friction or static friction* as it is usually called, and S_2 measures *the force of friction of motion* or *sliding friction.* If the pressure W between the surfaces be increased by adding weights to the block, a larger force, S_1, will be required to start the block than was needed before, and also a larger steady pull, S_2, to keep it sliding steadily. In all cases, however, S_1 will be greater than S_2, or *static friction is greater than sliding friction.*

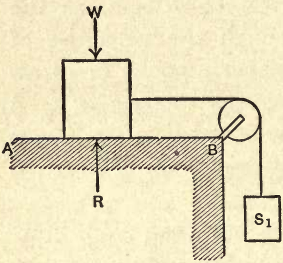

FIG. 145.

If a series of determinations of both sliding and static friction be made for different pressures between the surfaces in contact, the results may best be shown graphically, as in Fig. 146. The resulting curves will be straight lines passing through the origin. They show therefore that for

both static and sliding friction, the *force of friction is proportional to the pressure perpendicular to the surfaces in contact,* and therefore that:

$$\frac{\text{FRICTION}}{\text{PRESSURE}} = a \text{ constant.}$$

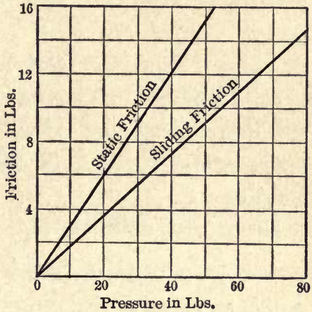

FIG. 146.

If we change the area of the surfaces in contact, *keeping the total pressure between them unchanged,* as for example, if we place *W* upon its narrow side, we will obtain practically the same values for the force of friction provided this side has been dressed in the same manner as the other. The force of friction is therefore practically independent of the area of the surfaces in contact, where cutting does not occur.

106. Coefficient of Friction.—The fraction, $\frac{\text{Friction}}{\text{Pressure}}$ is called the *Coefficient of friction.*

$$\frac{\text{Static friction}}{\text{Pressure normal to surface}} = \textit{Coefficient of friction of rest.}$$

$$\frac{\text{Sliding friction}}{\text{Pressure normal to surfaces}} = \textit{Coefficient of friction of motion.}$$

The coefficient of sliding friction, usually called simply the *coefficient of friction,* is the one commonly required in practice, and is represented by the symbol μ.

If instead of cast-iron surfaces we use brass on cast iron, cast iron on leather, oak on oak, fibers parallel, oak on oak, across the grain, etc., we will still obtain curves which are straight lines passing through the origin, but each curve will have a different slope. In other words, the fraction

$\frac{\text{Friction}}{\text{Pressure}}$ will be a constant for each case, but will have different values for each case. The coefficient of friction therefore expresses the effects of material, surface, etc., upon friction. The student will find it interesting here to compare the values of the coefficients of friction for different materials given in the larger reference books on mechanics.

From our definition $\mu = \frac{\text{Friction}}{\text{Pressure}}$, we see that, in general, FORCE TO SLIDE ONE BODY ON ANOTHER $= \mu \times$ NORMAL PRESSURE BETWEEN THE SURFACES.

107. Determination of Coefficient of Friction.—The coefficient of friction between two surfaces may be obtained

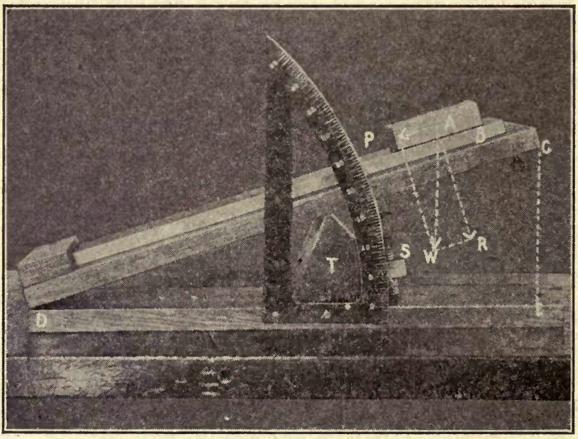

FIG. 147.—Laboratory Inclined Plane.

as in Fig. 145. A more convenient form of apparatus is shown in Fig. 147. *CD* is an inclined plane which can be adjusted to any desired angle of slope. Upon this are placed the surfaces to be tested, *A* and *B*, *A* being free to move. The weight, *W*, of *A* may be resolved into the two

forces P, tending to cause A to slide on B, and R, perpendicular to the line of contact between A and B, producing pressure between them.

Rough adjustment of CD is made by moving the triangular block T forward or back. Fine adjustment is then made by tipping the block by turning the screw S until, on tapping A to start it, it slides uniformly on B.

Thus friction $=P$ and $\mu=\frac{P}{R}=\frac{CE}{DE}=$ tangent of angle of plane CDE.

(Student should prove geometrically that $\mu=$ tangent of angle of plane.)

108. Friction of Machinery.—If in place of the apparatus shown in Fig. 145, a simple machine, as the wheel and axle of Fig. 148, be used, the value of E when W is raised steadily *must be enough to balance W and also overcome friction of the apparatus: When W is lowered steadily, E = force to balance W minus friction.* Calling force to balance W *on a frictionless machine P*, we thus have:

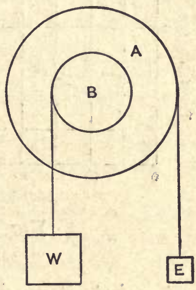

FIG. 148.

$$E, W \text{ rising} \qquad = P + Fr.$$

$$E, W \text{ descending} = P - Fr.$$

From these we see that friction, Fr

$$= \frac{\text{Difference between the two values of } E}{2}.$$

Plotting results obtained with different loads, W, will give the straight line of Fig. 149. This line gives an intercept K, on the Y axis, *equal to the friction of the machine unloaded*, and therefore its equation is, Friction = friction of unloaded machine + increase in friction due to load (equal to slope, $c, \times W$). Or,

$$(1) \qquad Fr = K + cW;$$

Or, substituting values from our curve,

$$(2) \qquad Fr = 1 + .05 \times \text{load}.$$

Equation 1 is a perfectly general expression for the friction in machines whether they are large or small, simple or complex. It should be noted, however, *that the value of friction is here measured at the circumference of the wheel.* Of course the actual resistance occurs in the bearings and is probably distributed, occurring at a number of points. It is entirely feasible, to measure it in terms of force required to turn the apparatus with a moment arm equal to the radius of the wheel. It might have been determined in terms of force applied to the circumference of the axle, or at any other convenient point, if so desired. Thus in machinery, a part of the driving force, wherever it is applied, is required to overcome the friction distributed through the machine, and the amount of force necessary for this purpose depends first upon the friction of the unloaded machine and secondly upon the increase in friction produced when load is put upon the parts. In general, therefore, friction is a disadvantage in machines, diminishing their *efficiency* as we shall see later. In a few special machines, as for instance a screw-jack, friction is necessary for their proper operation.

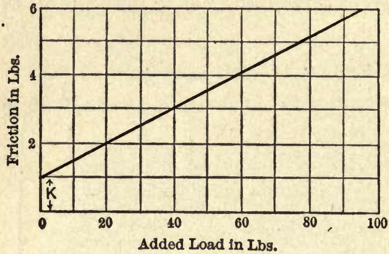

FIG. 149.

109. So-called Laws of Friction.—In the preceding articles, to make the points under discussion more clear, we have spoken of the friction of different surfaces, coefficients of friction, etc., as if they were fixed, and accurately

known values like the *constants* used in all science work. As a matter of fact, the factors controlling friction in any actual instance are so numerous, and so dependent upon variable conditions that only the most general principles may be stated positively. The following summary is taken from Perry's "Applied Mechanics."

LAWS OF FRICTION

FRICTION BETWEEN SOLIDS.	FLUID FRICTION.
1. The force of friction does not much depend on velocity but is greatest at slow speeds.	1. The force of friction very much depends on the velocity, and is indefinitely small when the speed is very slow.
2. The force of friction is proportional to the total pressure between the two surfaces.	2. The force of friction does not depend on the pressure.
3. The force of friction is independent of the areas of the rubbing surfaces.	3. The force of friction is proportional to the area of the wetted surfaces.
4. The force of friction depends very much on the nature of the rubbing surfaces, their roughness, etc.	4. The force of friction at moderate speeds does not much depend on the nature of the wetted surfaces.

From Perry's "Applied Mechanics," page 80.

110. Friction of Lubricated Surfaces.—The very great differences between the friction of dry bodies and of fluids, as stated in the preceding article, should be noted. Even with dry surfaces the amount of *air film* between the surfaces may greatly modify the friction. Lubricated surfaces, as ordinary bear-

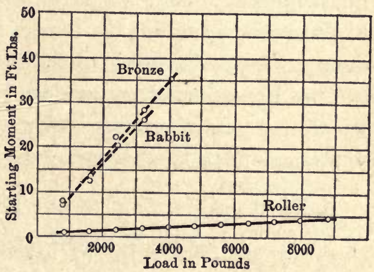

FIG. 150.—Starting Torque.

ings, etc., would naturally obey neither the laws for dry surfaces nor those for fluids, but would be intermediate between the two, varying from the case of *flooded bearings* which follow quite nearly the laws for fluids, to very modified and uncertain conditions as the degree of lubrication, kind of lubricant, temperature, pressure between surfaces, velocity, etc., change. No very definite statements can therefore be made for the friction of lubricated surfaces.

In general, the character of the lubricant should be such that it will be constantly dragged into the rubbing surfaces as needed. Where the pressure between the surfaces is great, light, thin oils would be forced out, hence for such cases heavy oils or grease are preferable. Under light pressures these would be too viscous.

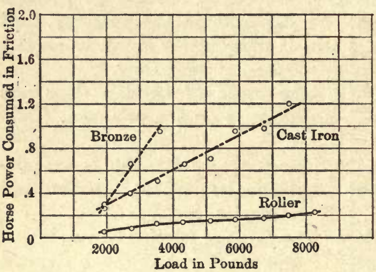

FIG. 151.—Horse Power consumed in Friction. Speed 130 revolutions per minute.

The properties of oils vary greatly with temperature also, and the selection of a lubricant must depend considerably upon the temperature to which it will be subjected. For high temperatures such as result when high-pressure steam is used, mineral oils which do not decompose with heat must be used.

The horsepower expended in driving a shaft = force of friction at the given speed $\times \pi \times$ diam. $\times$ r.p.m. $\div$ 33,000; or, what amounts to the same thing,

H.P. = Coef. friction at the given speed $\times$ pressure of shaft in bearings $\times \pi \times$ diam. $\times$ r.p.m $\div$ 33,000.

Poor alignment greatly increases the pressure of shaft in

its bearings and therefore the friction. Any computation based upon this equation, however, is necessarily only approximate owing to lack of knowledge of the friction at a particular speed. Figs. 151 and 152 show the results of tests of horsepower required to keep a shaft turning at constant speed in bronze, cast-iron and roller bearings at 130 r.p.m. and 460 r.p.m. with the same loads. At a speed approximately 3.5 times as fast, the horsepower for cast iron is only 2.5 and for roller bearings 2.9 times as much; or in other words, in these cases the friction is less at a high speed. In the case of the bronze bearing, on the contrary, the horsepower given by the test was 5 times as much, showing an apparent increase of friction with the speed. No divisions of mechanics furnish a better medium for instructive experiments than those dealing with the relation of friction to material, speed, lubrication, etc., and with the power required to drive a shaft under different conditions of lubrication, alignment, etc. The student should, therefore, be given ample opportunity for direct, practical tests upon a shaft driven by a variable speed motor. Friction brakes and transmission dynamometers for use in such tests are described in Art. 116.

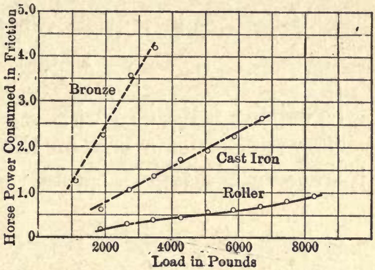

FIG. 152.—Horse Power consumed in Friction. Speed 460 revolutions per minute.

111. Friction of Belts.—Suppose a pulley P_1 is driving a second pulley P_2 by means of a belt, the direction of rotation being as shown by the arrows, Fig. 153; both sides of the belt, t and T, must be under tension in order to give pressure on the pulleys, and therefore friction which

shall keep the belt from slipping. But tension T must be greater than the tension t, otherwise P_2 will not turn. It is this *difference in tension* of the two sides of the belt therefore which measures the *force involved* in the transmission of power. *Difference in tension* $\times$ speed of belt in ft. per min. = work done per minute. And

$$\frac{\text{difference in tension} \times \text{speed in ft. per min.}}{33{,}000} = \text{H.P.}$$

transmitted from P_1 to P_2.

The friction of a belt on the pulley may be increased by increasing the tension—and therefore the pressure between the belt and pulley—or by increasing the arc of contact between the two. The usual practice is to drive with the *upper side* of the belt slack, so that any sagging due to the weight of the belt may increase the arc of contact. "Idlers" or tightening pulleys are often placed on the upper or slack side of the belt to increase both the tension and the arc of contact.

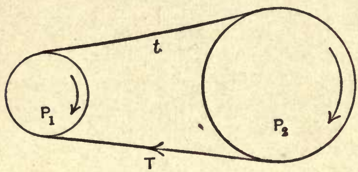

FIG. 153.

PROBLEMS

1. A block weighing 200 lbs. rests on a horizontal surface. It requires 90 lbs. pull to slide it along with uniform speed. Find the coefficient of friction and the work done in moving it 20 ft.

2. A stone is projected along smooth ice with a velocity of 15 ft. per second and comes to rest after traveling 75 yards. Find the coefficient of friction of stone on ice.

3. If it requires a force of 65 lbs. to move a body weighing 100 lbs. *uniformly* up a plane, inclined 30°, find coefficient of friction for the surfaces.

4. Find the work done in sliding a wooden body weighing 300 lbs. up an inclined plane 5 ft. high and 20 ft. long, the coefficient of friction being 0.4.

5. A block of stone is hauled along the ground by a pull of 310 lbs. at an angle of 10° above the horizontal. If coefficient of friction is 0.58, what is the weight of the stone?

6. A wagon weighing 2200 lbs. is pulled up a hill rising 2 ft. in 20 ft. of road. If the resistance is 12 lbs. per hundred weight, what pull parallel to the load must be exerted?

7. What is the *least* pull which will move a box weighing 200 lbs. along a concrete floor if the coefficient of friction is 0.50?

NOTE.—*Least* pull acts at an angle above horizontal whose tangent = coefficient of friction.

8. A ladder 30 ft. long weighing 100 lbs. rests against a wall. Center of gravity of ladder is $\frac{1}{3}$ the distance from the foot. Coefficient of friction with ground is 0.6 and with wall above is 0.3. Ladder makes angle of 45° with the wall. How far up the ladder can a man weighing 200 lbs. go before the ladder begins to slip?

9. A cast-iron fly-wheel has a rim 6 ft. mean diameter, 7 ins. wide, and 4 ins. thick. It is keyed to a shaft 4 ins. diameter, weighing 500 lbs. If the fly-wheel is running 148 revolutions per minute and the coefficient of friction between shaft and bearing is 0.05, approximately how many revolutions will the fly-wheel make in coming to rest?

10. What horsepower will be required to drive the loaded shaft of Problem 9, at the uniform speed of 240 revolutions per minute?

11. If the coefficient of friction of carbon on copper is 0.24 and if the average pressure of the 10 sq.in. of carbon upon a 14 in. diameter commutator of a motor is 3 lbs. per sq. in., what will be the horsepower used in overcoming friction at a speed of 600 revolutions per minute?

12. A N. Y. C. locomotive weighing 200 tons and exerting a tractive force of 20,000 lbs. hauls a trailing load of 400 tons. If friction is $6\frac{1}{2}$ lbs. per ton how long will the locomotive require to get up a speed of 55 miles per hour from rest: (*a*) On a level? (*b*) On a 2% grade?

13. What is the maximum trailing load that the locomotive of problem 12 can keep moving at a constant speed of 45 miles per hour up a 1% grade?

112. Law of Conservation of Energy.—This law, briefly stated, is:

> 1. Energy cannot be created or destroyed. The total sum of energy in the universe remains constant.
> 2. Energy may be transformed from one form to another; in such transformation no energy is ever lost.

These propositions have been established by observations and experiments extending over many years, and conducted by men working in all branches of natural science, especially in physics and chemistry. They are now universally accepted and form the fundamental principle for all theories not only in engineering, and physics and chemistry, but also in physiology, botany, astronomy, and in fact every natural science. The principle of the Conservation of Energy thus becomes both an aid to the understanding of many natural processes and a *test* by which science determines the reasonableness of each new theory or hypothesis.

The final, definite statement of the law of the conservation of energy dates from the middle of the nineteenth century, when it was experimentally proved that heat is a "form of energy," that is to say, A CERTAIN AMOUNT OF HEAT MAY BE MADE TO PERFORM A DEFINITE AMOUNT OF WORK, AND CONVERSELY A CERTAIN AMOUNT OF MECHANICAL WORK, IF TRANSFORMED INTO HEAT, WILL FURNISH A

DEFINITE AMOUNT OF HEAT. This fact may, for our purpose, be stated thus:

THE ENERGY REQUIRED TO HEAT 1 LB. OF WATER ONE DEGREE FAHRENHEIT, IS THE SAME AS THAT REQUIRED TO PERFORM 778 FT.-LBS. OF WORK.

Of course the equivalence of various "forms of energy" may be stated in various other ways, which will include such apparently different ideas as electrical and chemical relations and the phenomena of sound, light, and animal and vegetable processes. Thus take 1000 ft.-lbs. of mechanical energy and by proper mechanism, either natural or of human devising, transform it into other forms of energy; we find that always a certain perfectly definite amount of chemical or electrical or other energy is obtained and that by no device can we ever produce more energy than the 1000 ft.-lbs. we started with. It is perhaps unnecessary to add that the complete tracing of these transformations and the measurement of the amounts at all stages of the processes is not humanly possible, since the processes are as intricate and manifold as nature itself. All our experiments, however, point undeniably to the truth of the great fact that energy may be transformed but never destroyed.

It must not be inferred that the law of conservation of energy means that the supply of energy *available for our use* is constant. In all machinery by which energy is utilized, there is a constant loss of useful energy through the transformation into heat energy by friction at bearings, etc., and the dissipation of this heat. Thus, as an illustration, suppose 10,000 units of heat energy are liberated by the combustion of 1 lb. of coal in the boiler furnace. Of this about 7500 units are given to the steam, the other 2500 units are dissipated through the ash, chimney gases, radiation, etc. Of the 7500 units carried into the cylinder

about 800 units are transformed into mechanical energy of the moving piston, 6700 units being lost through condensation, radiation, etc., in the cylinder, or carried on into the condenser. If the engine is driving a dynamo, about 700 of the 800 units may reappear in the energy of the electric current, the other 100 disappearing as heat through friction, electrical losses, etc. If the dynamo is supplying arc or incandescent lights, the 700 units are given to heat the connecting wires, lamp filaments, etc., and thus all our original supply of energy has been dissipated. Or, if the dynamo drives a motor, about 650 units may be obtained in mechanical energy supplied by the motor, 50 being lost in the transformation through friction, electrical losses, etc. Of the total 10,000 units with which we started, therefore, approximately 94.5% have been dissipated, 6.5% are still available for doing work.

Many similar examples might be given. In all we find that each step in the process of utilizing energy is accompanied by unavoidable loss.

113. Machines.—For students of Mechanics the immediate and most useful application of the Principle of the Conservation of Energy is in the study of machines. In the most general terms we may say that, *A machine is a device by which energy received from some source outside itself is delivered at some other point, after certain "losses" and transformations, there to perform some special work.*

Thus with a derrick, the energy supplied to the hoist serves to lift weights against gravity. An engine is a machine in which the heat energy of the steam is transformed, with certain "losses," into the kinetic energy of rotation of the fly wheel.

It will be seen that the law of the conservation of energy furnishes us with certain necessary specifications for the complete study of any machine. From such study we may

arrive at a determination of a machine's efficiency, that is, of the per cent. of the energy supplied that is actually employed in the useful work for which the machine is designed.

At the driving end a more or less constant stream of energy is supplied from the driving belt, shafting, moving water, steam, electric motor, etc. None of this energy received, or "*input*," can be destroyed; that which does not pass through and reappear in the *output* of the machine, has been required for internal losses which have dissipated it as non-recovered heat. Each moving part of the machine plays its part in this transmission of energy. Each has its own store of energy which is constantly augmented by the supply received from the preceding part of the mechanism, and diminished by the energy handed on to the next following part with which it is joined. Starting at the driving end, a debit and credit sheet might be kept for each succeeding unit in the mechanism; and in this every part would be found *unable to repay the energy received in full because of the inevitable operating losses.*

The designer may *reduce the waste* of energy at any particular point by devices for lessening friction, and by properly proportioned bearings, etc., which may diminish the distance through which friction must be overcome. He may modify a fluctuating energy supply into a comparatively steady output by fly-wheels attached to certain parts, which on account of their great capacity for storing kinetic energy, absorb a great supply without great increase in speed, and then when the supply temporarily diminishes or great demand is suddenly made on the part, can supply a large quantity of energy without greatly slowing up; or he may attach a governor to a steam engine or a water wheel to automatically adjust the supply of energy to meet the varying demands for energy made by the machinery in operation. But for any form of machine,

whether simply mechanical or involving the use of steam, water, electrical or other motive power, he must work in accordance with the principle of the conservation of energy, for he cannot supplant it. He must allow for the inevitable losses, realizing that not all of the energy supplied can be delivered for performing useful work.

Thus machine design becomes primarily a study of energy relations and energy losses. The "explanation" of the operation of any machine involves a recognition of the energy supply, the mechanism and processes of energy transformations, sources of energy loss, and the fact that the energy finally delivered for the purpose desired is only a fraction of the energy received.

114. Important Terms Applied to Machines.—The following definitions are important in connection with machines:

1. INPUT.—*The energy supplied* TO *the machine is known as the* INPUT.

2. OUTPUT.—*The energy delivered* BY *the machine for the purposes for which it was designed is known as the* OUTPUT.

3. EFFICIENCY.—*The efficiency of a machine equals the ratio of output to input.* It is usually expressed as a per cent, i.e., per cent of input which is delivered by the machine. Thus:

$$\text{Efficiency} = \frac{\text{output}}{\text{input}},$$

and

$$\text{Output} = \text{input} \times \text{efficiency}.$$

4. VELOCITY RATIO.—*The distance through which the driving force acts divided by the distance through which the force exerted by the machine acts in the same time, is known as the* VELOCITY RATIO.

Thus referring to the simple wheel and axle shown in Fig. 148, if circumference of wheel A is 48 ins. and of axle B is 15 ins., and the apparatus is used for hoisting loads applied at W, for each revolution the force applied E moves 48 ins., the force exerted by machine, or W, moves 15 ins.; the velocity ratio is thus $\frac{48}{15}=3.2$.

The velocity ratio is thus a question of the design of the machine.

5. MECHANICAL ADVANTAGE.—*The ratio of force exerted by a machine to driving force is known as the* MECHANICAL ADVANTAGE *of the machine.*

Thus, referring again to Fig. 148,

$$\text{Mechanical advantage}=\frac{W}{E}.$$

It is obvious that if the machine could be *frictionless output would equal input*, and work done on W would equal work done by E, or $W\times$circumference of $B=E\times$circumference of A. Therefore, $\frac{W}{E}=\frac{48}{15}=3.2$, or mechanical advantage would equal the velocity ratio. In any *actual machine*, however, because of friction, the driving force must be increased from that required for a frictionless machine, hence *the actual mechanical advantage will always be less than the velocity ratio.* It will also be variable with the conditions of lubrication, bearing surfaces, etc., and must therefore be determined by actual experiment at the time the machine is used.

115. General Law of Machines.—From the preceding discussion it will be seen that the general law for any type of machine will be of the form:

Input = output + energy expended against friction.

Or,

$$E \times d_1 = (L \times d_2) + (Fr + d_3).$$

Where E = driving force, L = force applied by machine —commonly called *Load*—Fr = force to overcome friction, and d_1, d_2, d_3 are the distances through which driving force, load, and friction act, respectively.

Assuming no friction, as is sometimes convenient, this equation becomes,

$$\text{Input} = \text{output} \quad \text{or} \quad E \times d_1 = L \times d_2.$$

In the study of any actual machine where friction is always present and therefore some of the input is not recoverable, a convenient expression of the law is:

$$\textit{Input} \times \textit{efficiency} = \textit{output},$$

or

$$E \times d_1 \times \text{efficiency} = L \times d_2.$$

116a. Dynamometers.—Instruments for measuring power are commonly called *dynamometers*. These are, generally speaking, of two classes: (1) *absorption dynamometers* which absorb the power received, transforming it into heat; and (2) *transmission dynamometers* which absorb only power enough to operate their mechanism and transmit the remainder.

Dynamometers of the first type are used where the *output of a source of power* is to be measured, as for example, the power delivered at the belt pulley of a motor, engine, etc. This is termed the *brake horsepower* of the motor or engine.

Those of the second type are inserted *between* the source of power and the machine being driven to determine the power delivered to any particular belt, shaft, etc., in the

transmission line. Transmission dynamometers thus enable us to determine the power lost at any stage in the power transmission, through friction, belt slip, etc.

116b. Prony Brakes.—The simplest and most commonly used form of absorption dynamometer is the *prony brake.* This may be of several different forms, all of which are based upon the same fundamental principle. Fig. 154 shows a common form, where A indicates the belt pulley of a motor, engine, etc. This is surrounded by the brake band B, which may be tightened as desired by means of the screw at C. As the pulley revolves in the direction of the arrow, friction tends to carry the band around also, but this tendency is counteracted by a force W acting on a lever arm L attached to the band.

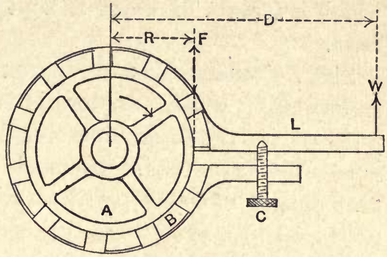

FIG. 154.—Prony Brake.

If F represents the force of friction,

$$F \times R = W \times D,$$

and
$$F = \frac{W \times D}{R}.$$

The work in ft.-lbs. *absorbed by the brake in one revolution* is equal to friction in lbs. times the distance in feet through which it is overcome, or

$$F \times \text{circumference of pulley.}$$

And the power absorbed is therefore,

$$\frac{F \times \text{circumference of pulley} \times \text{revolutions per minute}}{33{,}000}.$$

The friction brake evidently transforms the mechanical energy delivered into heat. The great amount of heat thus generated usually makes it necessary to use a cooling device with the brake. Pulleys with hollow rims which may be filled with water, or hollow brake straps through which a stream of water is kept flowing are therefore often used.

116c. Transmission Dynamometers.—The object in all transmission dynamometers is to absorb as little power as possible in the dynamometer itself and to measure the amount transmitted. In most common types, this measurement is effected by determining either the extension produced in a system of springs attached to the device, or the torque required to keep in position a frame which carries a system of pulleys on which the driving belts run.

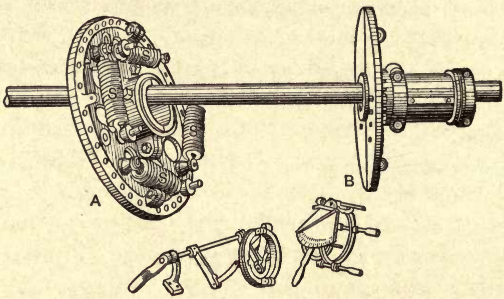

FIG. 155.—Transmission Dynamometer.

The principle of the former will be apparent from the diagram of Fig. 155. This shows a simple dynamometer coupling in which connection between two parts of a shaft is made by the springs *SSS* instead of bolts or pins, as in the ordinary rigid coupling. Plates *A* and *B* are each keyed to the shaft. As the side *B* is driven, the torque is communicated to *A* through the springs which are thereby

elongated. If, now, the *constant of the springs* is known, also the *amount they are extended*, and *the speed* in revolutions per minute, the horsepower transmitted by the shaft may be readily computed.

The constant for the springs may be found experimentally by holding A and applying a known torque to B by means of cords and weights. Suppose, for example, a torque of 25 lb.-ft. causes the springs to extend 2.5 scale divisions; every scale division then represents 10 lb.-ft. of torque or $10 \times 2 \times \pi = 20\pi$ ft.-lbs. of work per revolution.

If, therefore, in an actual instance, the reading on the dynamometer scale is 10.6 divisions at 300 r.p.m., the horsepower transmitted is,

$$\text{H.P.} = \frac{10.6 \times 20\pi \times 300}{33{,}000} = 6.06.$$

The practical difficulty is, of course, to transmit the extension of the springs to an index which may be read while the shaft is rotating, and in the device by which this is accomplished lies the main difference between the instruments in use.

In the *Webber dynamometer* the change in the relative angular positions of the plates operates levers which move a loose sleeve on the shaft. This sleeve in turn operates an index moving over a graduated dial which gives the horsepower.

In the *Emerson Power Scale* the springs are replaced by a weighing device somewhat similar in principle to a steel yard. A and B are connected by projecting studs. The pressure on these studs is conveyed by levers to a collar which in turn is connected to a *weighing lever* where the force is measured by balancing weights. Sudden motion of the weighing lever is prevented by dash-pots.

The preceding types are adapted to use with a shaft.

By the addition of a pulley to the shaft to which A is keyed, they may also be adapted to measure the power transmitted by a belt. Two belts must be used in place of one, the shaft or machine being driven *through the dynamometer*. The belt from the motor or engine drives B, and B drives A through the connecting springs or studs. A second belt runs from the extra pulley keyed to the shaft of A, to the machine to be driven. Computation, devices for reading, etc., are the same as before.

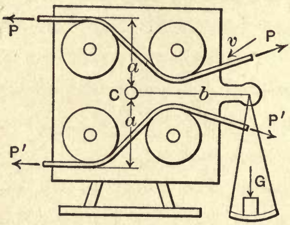

FIG. 156.—Belt Dynamometer.

A second type of transmission dynamometer for use with a belt is shown in Fig. 156 (see Church's "Mechanics of Materials"). A vertical plate carrying four pulleys upon which the driving belt runs is balanced about a pivot at C. Power is then turned on, and the plate again balanced. If G = weight to balance,

$$Gb = Pa + P'a,$$

(P and P' on right pass through C and therefore have no moment).

Therefore, $$P - P' = \frac{Gb}{a}.$$

$$\text{Work per minute} = (P - P')v,$$

when v is velocity of belt in feet per minute.

For a more detailed discussion of particular types of transmission dynamometers, the student is referred to engineering texts, trade periodicals, etc.

117. Applications of the Preceding Principles to Simple Machines.—The study of mechanism is regarded as entirely

outside the scope of this book. The preceding discussion, together with the solutions which follow, are intended to familiarize the student with some of the more general technical terms and computations which apply to machinery, and above all, to enable him to approach the formal study of mechanism later with a right point of view. A complete and formal discussion of what are generally termed the "Simple Mechanical Principles," has been purposely avoided.

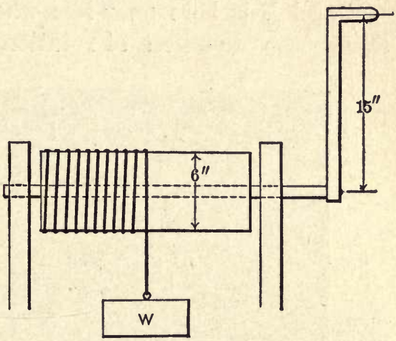

FIG. 157.—Winch.

118. The Winch.—Neglecting friction, what load can be lifted by a driving force of 50 lbs. applied at right angles to the crank of the winch shown in Fig. 157?

In one revolution, driving force moves through the distance equal to the circumference of the circle swept out by the crank, or

$$\frac{15}{12}\times 2\times\pi=\frac{15}{6}\pi \text{ ft.}$$

In the same time load is lifted a distance equal to the circumference of axle $=\frac{1}{2}\times\pi=\frac{\pi}{2}$ ft.

$$\text{Velocity ratio therefore}=\frac{\frac{15}{6}\pi}{\frac{1}{2}\pi}=5,$$

or driving force moves 5 ft. to lift L 1 foot. Therefore,

$$50\times 5=L\times 1 \quad \text{and} \quad L=250 \text{ lbs.}$$

119. Gear, Sprocket, and Belt Machines.—In a system of gears, as shown in Fig. 158, A has 80 teeth, B has 60, and C has 36. Diameter of drum $D_1=3$ ins., of $D_2=9$ ins. Load $L=200$ lbs. Efficiency of system for this load $=80\%$. What driving force will be required at E?

Wheel B is here an "idle wheel," and serves merely to change the direction of rotation of C. The teeth prevent

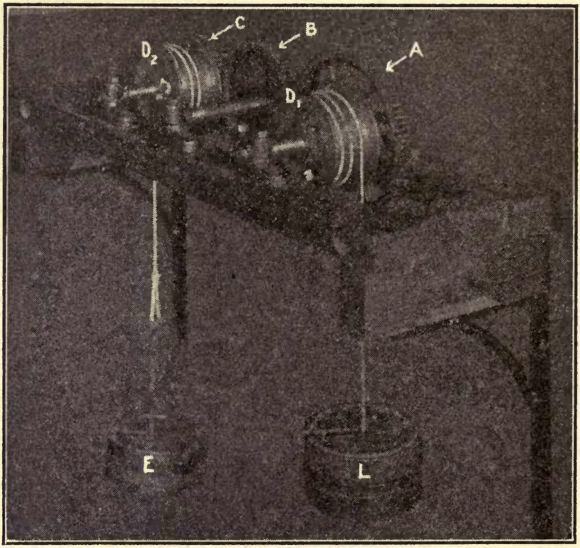

FIG. 158.—Gearing arranged for Laboratory Test.

slipping, hence linear speed of teeth on A, B, and C must be the same, and the speed of C will be the same as if A were geared directly on C. In one revolution of C, E moves the circumference of $D_2=9\pi$ ins.

When C makes one revolution, the same number of teeth pass the point of contact on A, or A makes $\frac{36}{80}$ of a revolution. L is therefore lifted

$$3\pi\times\frac{36}{80}=\frac{27}{20}\pi \text{ ins.}$$

$$\text{Velocity ratio}=\frac{9\pi}{\frac{27}{20}\pi}=6\tfrac{2}{3},$$

or E moves $6\frac{2}{3}$ ins. to raise L 1 in. Therefore,

$$E\times 6\tfrac{2}{3}\times\frac{80}{100}=200\times 1,$$

and

$$E=37.5 \text{ lbs.}$$

Fig. 159 shows an arrangement for testing the efficiency of transmission of the chain drive of a simple bicycle.

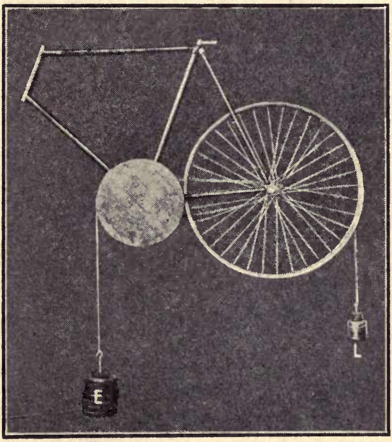

FIG. 159.—Bicycle Chain Drive arranged for Laboratory Test.

The pedals have been removed and a wooden pulley having a radius equal to the length of the crank has been attached to one crank. A pound applied to the rim of this pulley thus produces the same torque as an equal force formerly did when applied to the pedal. A resisting torque is applied to the rear wheel by a weight on the cord running on the rim, from which the tire has been removed. If desired, the machine may now be driven by the force on

the rear wheel. This will raise a greater weight on the pulley, thus gaining a large "mechanical advantage." Or it may be driven in the usual way as a "speed" machine with the force on the pulley used as the driving effort.

A two-ounce load is hung from the rear wheel and then enough weight put on the pulley to just raise this at a uniform speed. Then weights are removed from E until the load L lowers at the same uniform rate. Half the difference between these values (effort to raise and effort to lower the load) represents the friction of the machine at this load. The driving effort for a frictionless machine can be found by taking half their sum. The input, output, and efficiency are computed and curves plotted to show the relation of efficiency and driving effort to load raised. Also on the same sheet the driving efforts for a frictionless machine are plotted against the loads. The increase of friction with added loads is brought out by the divergence of the two "effort" curves. The displacement ratio is computed by measuring the diameters of the pulley and wheel, and counting the number of teeth on each sprocket. The computed value is then tested by actual measurements of the distances moved over simultaneously by effort and load.

Below is the report of two students, showing results of a test with this apparatus:

THE CHAIN DRIVE

DATA

Diameter of driver, 12.0 ins.
Diameter of follower, 25.0 ins.
Number of teeth on front sprocket, 18.
Number of teeth on rear sprocket, 7.

$$\text{Displacement ratio} = \frac{\text{driver}}{\text{follower}} \times \frac{\text{driver}'}{\text{follower}'} = \frac{12}{25} \times \frac{7}{18} = 0.187.$$

Displacement ratio =

$$\frac{\text{Distance moved by effort, 6.7 ins.}}{\text{Distance moved by load, 36.0 ins.}} = 0.19.$$

Displacement ratio (computed) = 0.187.
Displacement ratio (checked) = 0.19.

Load in Lbs.	Effort to Lift Loads. Lbs.	Effort to Lower Load. Lbs.	Effort for Frictionless Machine.	Effort to Overcome Friction.	Input, Ft. Lbs.	Output. Ft. Lbs.	Per Cent Efficiency
DATA.				COMPUTED VALUES.			
.13	.94	.63	.78	.16	.18	.13	72
.25	1.59	1.28	1.44	.16	.30	.25	83
.50	2.90	2.56	2.73	.17	.551	.50	91
1.00	5.60	5.18	5.39	.21	1.06	1.00	94
1.50	8.38	7.75	8.02	.32	1.59	1.50	94
2.00	11.0	10.3	10.7	.35	2.09	2.00	96
2.75	15.0	14.1	14.6	.45	2.85	2.75	97
3.75	20.5	19.3	19.9	.60	3.89	3.75	97
5.75	31.2	29.9	30.6	.65	5.93	5.75	97
6.88	37.4	36.0	36.7	.70	7.10	6.88	97

The curve plotted between driving efforts and loads shows that the *increase* in driving effort is directly proportional to any *increase* in load. The curve does not pass through the origin because it requires some effort to drive the machine unloaded; the "y" intercept shows this effort to be $\frac{1}{4}$ lb.

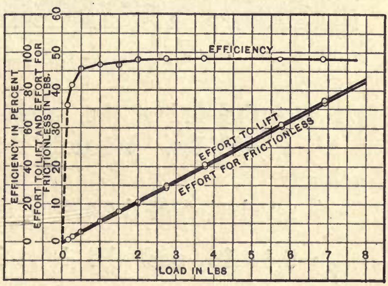

FIG. 160.

The curve between loads and efforts for a frictionless machine shows that these efforts are directly proportional to the loads themselves. This curve passes through the origin because no effort would be required to drive an unloaded, frictionless machine.

The curve between efficiency and loads shows that, within the limits of the experiment, the efficiency at first increases as the load increases, until, at a load of $2\frac{3}{4}$ lbs., the efficiency becomes practically constant at 97%.

120. The Jack Screw.—A jack screw is turned by a lever $1\frac{1}{2}$ ft. long from center of screw. There are six threads to the inch on the screw. A force of 100 lbs.,

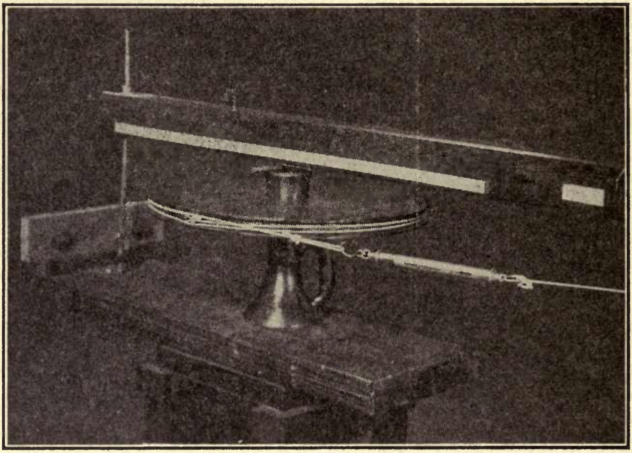

FIG. 161.—Jack Screw equipped for Laboratory Test.

applied at the end of the lever and at right angles to it, will lift 25,400 lbs. What is the efficiency of the jack at this load?

Each revolution of the screw, the driving force moves $2\times\frac{3}{2}\times\frac{22}{7}=9.42$ ft., and the load is lifted distance between threads, or $\frac{1}{6}$ in. $=\frac{1}{72}$ ft. Therefore,

$$100\times 9.42\times \text{efficiency}=25{,}400\times\tfrac{1}{72},$$

and $$\text{Efficiency}=37.6\%.$$

The apparatus shown in Fig. 161 is arranged for the test of a small jack-screw. No attempt is made to introduce any refinements into either the apparatus or the methods of making readings, but the purpose has been to retain, as nearly as possible, the actual working conditions obtaining in practice.

The jack-screw is a small one having a head upon the screw spindle, turning on roller bearings. The screw has a pitch of $\frac{3}{8}$ in. The base rests upon a heavy plank suitably supported. To the head of the screw is fastened a grooved wooden wheel, and the screw is turned by spring balances attached to cords wrapped about this wheel.

Across the head rests a round steel rod, and upon this rod rests a substantial wooden lever about 9 ft. long. This lever has one end fastened down to the floor by an iron rod provided with a thread and nuts at the top, so that, as the screw is used at different portions of its length, the lever may always be kept horizontal. A scale is attached to the lever, so that the distance of joint A from the lever fulcrum at B may be read easily. (See Fig. 12.) By shifting the jack-screw along the floor the ratio of the lever arms may be varied within certain limits.

The weight of the screw, head and wheel are determined and also that of the lever. The center of gravity of the lever is found by balancing it across a knife edge and its position is marked on the lever. At C is a clevis and scale-pan for supporting the variable loads as desired.

The methods used in calculation are as follows (see Fig. 12):

To find the vertical pressure on the screw spindle for any weight, L, at C.

$$\text{Pressure} = (\text{weight of wheel, etc.}) + \left(\text{weight of lever} \times \frac{AD}{AB}\right) + \left(L \times \frac{AC}{AB}\right).$$

In the test reported below this gives:

$$\text{Pressure at } B = 18.5 + \left(21.5 \times \frac{44.75}{18}\right) + \frac{98.9}{18} L = 72 + 5.5L.$$

Velocity Ratio: In one revolution of the wheel the *resistance* at B is overcome through a distance equal to the pitch of the screw, or $\frac{3}{8}$ in.

The *effort,* applied to the rim of the wheel, will in one revolution move a distance equal to the circumference of the wheel, or $20 \times 3.14 = 62.8$ ins. Therefore the *velocity ratio* is

$$\frac{62.8}{\frac{3}{8}} = 167.4.$$

This is equivalent to saying that for one foot moved by the weight at B the effort on rim of wheel must move 167.4 ft. These figures are the ones used in the calculation of input and output in the table below.

To find the force of friction and the effort that would be required if there were no friction, we proceed as follows:

Let E = effort with no friction;
Fr = friction;
S_1 = effort for a rising load at C;
S_2 = effort for a descending load at C.

NOTE.—All these four quantities are here understood to be the force in pounds at the rim of the wheel.

Then E must be increased by Fr to give the force to lift the load, or

$$(a) \qquad S_1 = E + Fr.$$

The machine will not run back of itself, but a force must be applied to turn it back; this force is S_2. Then

$$S_2+E=Fr,$$

or

(b) $$S_2=Fr-E.$$

Adding (a) and (b)

(c) $$\frac{S_1+S_2}{2}=Fr.$$

Subtracting (b) from (a)

(d) $$\frac{S_1-S_2}{2}=E.$$

Since the velocity ratio is 167.4, $E\times 167.4$ should give the pressure at B.

In the results of student tests given below, the loads were determined by weighing L on a platform balance. The efforts at rim of wheel were found by spring balances, a four pound balance reading to ounces for the lower reading and a larger balance for the higher ones.

Weight at C. (L Lbs.)	Total Load at B, $P=72+5.5\ L$ Lbs.	Efforts at Rim of Wheel.				Input $S_1\times 167.4$, Ft.-Lbs.	Output $P\times 1$, Ft.-Lbs.	Efficiency, Per Cent.
		For Rising Load, S_1 Lbs.	For Descending Load, S_2 Lbs.	Effort for no Friction, E Lbs.	Friction, Fr Lbs.			
0	72	1.19	.25	.47	.72	199	72	36.2
8.29	117	1.81	.38	.71	1.09	302	117	38.8
19.1	178	2.75	.63	1.06	1.69	460	178	38.8
40.6	229	4.25	.75	1.75	2.50	710	299	42.0
60.6	406	6.00	.94	2.53	3.47	1000	406	40.6
80.6	515	7.50	1.75	2.88	4.63	1250	515	41.2
102.1	632	9.50	2.00	3.75	5.75	1590	632	39.8
122.3	742	11.00	2.25	4.37	6.62	1840	742	40.3
132.2	800	12.00	2.50	4.75	7.25	2000	800	40.0

It is apparent from the table that the efficiency is nearly the same for all the loads used, though there is a slight increase for the larger loads. If still larger loads were used there might be more variation in the efficiency.

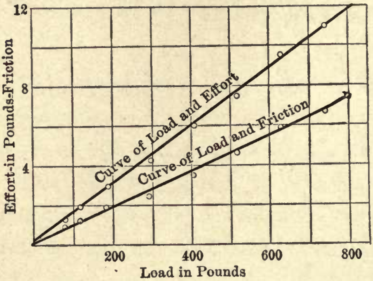

FIG. 162.

The results of this test are also shown in the plotted chart, see Fig. 162, where total load, P, and effort for a rising load, S, are plotted and also total load and friction. It appears here that both the effort and the friction are proportional to the total load within the range of loads used.

121. The Block and Tackle.—In the pulley blocks shown in Fig. 163, suppose L were lifted 1 in., then each of the five cords supporting L would be shortened 1 in. and to keep the rope tight E must move 5 ins.

Velocity ratio therefore $=5=$ number of parts of rope *supporting movable block.*

122. The Chain Hoist.—The Weston differential pulley, used for hoisting loads, consists of *two sheaves of different diameters in the upper block,* rigidly fastened together, and one sheave in the lower block. An endless chain runs over these blocks, the rims of the sheaves being formed so as to prevent the chain from slipping upon them. The arrangement is shown in Fig. 164. Suppose R be radius of larger sheave, r radius of smaller. To hoist load L, the driving force E is applied to chain coming from larger sheave as shown. Suppose upper pair of sheaves make one revolution. Then E moves a distance $2\pi R$, side B of chain is raised a distance $2\pi R$, but side A is *lowered* at the

same time a distance $2\pi r$, as it unwinds from the smaller sheaves. The loop of chain AB is actually shortened, therefore, a distance $2\pi R - 2\pi r = 2\pi(R-r)$. The load L is raised one-half of this distance, or $\pi(R-r)$.

$$\text{Velocity ratio} = \frac{2\pi R}{\pi(R-r)} = \frac{2R}{R-r}.$$

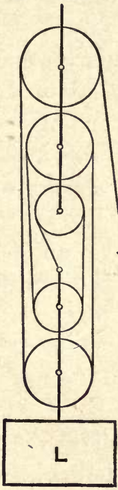

FIG. 163.—Block and Tackle.

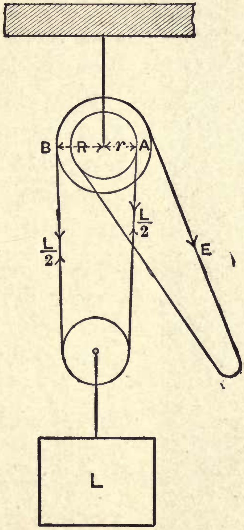

FIG. 164.—Weston Differential Pulley.

By making the sheaves nearly the same size (i.e., $R-r$ small), this may be increased.

The energy equation for the apparatus would be,

$$E \times 2R \times \text{efficiency} = L \times (R-r).$$

As used this pulley does not *run down* when force E is

removed. The tendency to turn clockwise, as will be seen from the figure, is $\frac{L}{2}\times r$. The counter-clockwise moment is $\frac{L}{2}\times R$. The counter-clockwise moment therefore exceeds the clockwise by the difference, or $\frac{L}{2}(R-r)$. *Friction of the apparatus* (upper and lower blocks) *must exceed this* and then the load will not run down.

FIG. 165.—Quarter-ton Chain Hoist Loaded.

The chain hoist furnishes an excellent medium for a laboratory study of a "non-reversible" machine. An arrangement of apparatus for such a test of a quarter-ton hoist with 8-ft. lift, is shown in Fig. 165.

The block is fastened to an I-beam. The load consists of an iron cradle and nine 50-lb. weights which may be added one at a time in order to test the machine under different loads from 50 to 550 lbs.

The procedure is as follows: The diameters of the differential pulleys are first measured and the displacement ratio computed from the formula $\left(\frac{2R}{R-r}\right)$, where R is radius of large pulley and r the radius of the smaller one. This is then checked by measuring the length of chain running over large pulley, which has to be displaced in order to raise the load one foot.

The effort required to raise a given load is measured

by pulling on a spring balance attached to the chain. As the machine will not run backwards, a negative effort, i.e., an effort on the other side of same pulley, must be applied to lower the load. The amount of negative effort is determined in the same way as the effort required to raise the load. Half the algebraic difference of these two quantities is approximately the value of the friction at that load. The output is computed by assuming load to be lifted 1 ft., thus: Output = load × 1 ft.-lbs. The input then equals the product of the displacement ratio, by the effort to raise the load. By dividing output by input, the efficiency is found at the given load. This is repeated for loads increasing by 50-lb. steps until a load of about 550 lbs. is reached. Curves are then plotted, showing relation between load and effort, load and friction, and load and efficiency.

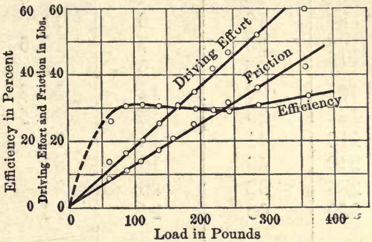

FIG. 166.—Relation of Efficiency, Driving Effort, and Function to Load in Chain Hoist.

Below is a report of two students on this apparatus.

THE CHAIN HOIST

Radius large pulley $R = 4.5$ ins.
Radius small pulley $r = 4.0$ ins.

$$\text{Displacement ratio} = \frac{2R}{R-r} = \frac{2 \times 4.5}{4.5-4.0} = 18.$$

Check of displacement ratio =

$$\frac{\text{distance effort moves}}{\text{distance load moves}} = \frac{72 \text{ ins.}}{4 \text{ ins.}} = 18.$$

The curves, Fig. 166, plotted from this data show an intercept on the Y-axis of ½ lb. This is the driving effort

to overcome friction with no load. The efficiency increases to about 30% at 75 lbs. load and then remains practically constant within the limits of the test.

Data.			Computed Resuts.			
Load in Lbs.	Effort to Raise Load, Lbs.	Effort to Lower Load, Lbs.	Friction, Lbs.	Input, Ft.-Lbs.	Output, Ft.-Lbs.	Efficiency Per Cent.
61	13	4	8.5	234	61	26
87	16	6	11	288	87	30
108	20	7	13.5	360	108	30
137	25	9	17	450	137	30
165	30	11	20.5	540	165	31
191	36	13	24.5	648	191	30
216	41	15	38	738	216	29
240	46	16	31	828	240	29
286	52	20	36	936	286	31
359	60	24	42	1080	359	33

123. Energy in Starting and Stopping Machines.—In the act of starting any machine under a given load a greater supply of energy must be furnished than is required to maintain the motion at a uniform speed when started, because every moving part must be *accelerated* from rest up to some velocity. Similarly a machine in motion has stored in its parts a supply of kinetic energy $\left(\frac{WV^2}{2g} \text{ ft.-lbs.}\right)$, which will be available to do some work even after the actual driving power ceases to act. (If the force of friction is large the *useful* work done may be so small as to be not very apparent.) If the throttle of a locomotive be closed after the locomotive is set in motion, the kinetic energy of the locomotive and train may suffice to carry them some distance along the track against the resistance of friction and perhaps of gravity also on an up grade. Furthermore it should be noted that the kinetic energy of

all the rotating wheels is available as well as the energy of the train along the rails. All this kinetic energy had to be supplied to the train either by the engine or gravity (on a down grade), or both, in the act of starting the train from rest, and remains stored until in stopping it is finally dissipated, chiefly as heat.

PROBLEMS

1. A safe weighing 5 tons is to be loaded on a truck 4 ft. high by means of planks 18 ft. long. If it requires 200 lbs. to overcome friction of the safe on the planks, find least force which will be necessary.

2. A safety valve is arranged as in Fig. 167. Weight of $AB = 12$ lbs., weight acts 10 ins. from A. Weight of valve $V = 8$ lbs. Diameter of valve 3 ins. What effective steam pressure per square inch will just open the valve?

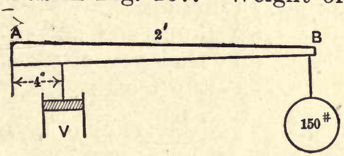

Fig. 167.

3. In a letter press, the diameter of the wheel is 14 ins., pitch of thread is $\frac{3}{16}$ in. Find pressure applied by a pull of 50 lbs. tangent to the wheel if the efficiency of the press is 30%.

4. In an ordinary "jack-screw," screw is turned by a lever 2 ft. long (from center of screw). If there are 4 threads to the inch, find force required on the end of the lever to lift 10,000 lbs. when 65% of the total effort is required to overcome friction.

5. If an effort of 100 lbs. acting upon a machine moves with a velocity of 10 ft. per second:

(*a*) How great a load can it give a velocity of 125 ft. per second?

(*b*) With what velocity can it move a load of 200 lbs.?

6. The pilot wheel of a boat is 3 ft. in diameter; the axle is 6 ins. The resistance of the rudder is 180 lbs. at the axle. What effort applied to the wheel will move the rudder?

7. Four men are hoisting an anchor weighing one ton. Barrel of capstan is 8 ins. diameter, length of hand spikes 3 ft. 4 in. (from center of barrel). How great pressure must each man exert? What is the velocity ratio?

8. In moving a building, the horse is attached to a lever 7 ft. long, acting on a capstan barrel 11 ins. diameter; on the barrel winds a rope of a system of 2 fixed (near capstan) and 3 movable pulleys. What force will be exerted when the horse pulls 500 lbs., allowing 50% loss from friction?

9. In a simple wire-testing machine the displacement ratio is 480. An effort of 1.6 lbs. will support a load of 32 lbs. Compute:

(*a*) Input and output in moving load 2 ins.
(*b*) Efficiency of machine at this load.
(*c*) Effort expended in overcoming friction.

10. A railway engine hauls a train on a level track at a speed of 30 miles per hour against an average resistance of 4500 lbs. It burns 760 lbs. of coal per hour. If the combustion of each pound of coal supplies 10,000,000 ft.-lbs. of energy, compute input, output, and efficiency of the engine.

11. Diameter of wheel A, Fig. 168, is 16 ins.;
" " " B, 10 ins.;
" " drum C, 12 "
" " " D, 3 "

If 5 lbs. are required at E to overcome friction when $L = 90$ lbs., what value of E will raise L uniformly? What is efficiency of apparatus at this load?

12. By experiment with a Weston pulley it was found that a pull of 15 lbs. on the leading side was required to lift 60 lbs., including lower pulley. The larger pulley was 4 ins. diameter, the smaller, $3\frac{1}{2}$ ins. Find efficiency at this load. (See Fig. 164.)

13. In the worm and worm-wheel, Fig. 169, wheel A has 98 teeth, and drum B is 8 ins. diameter. The worm is turned by a handle 18 ins. long. What is the velocity ratio?

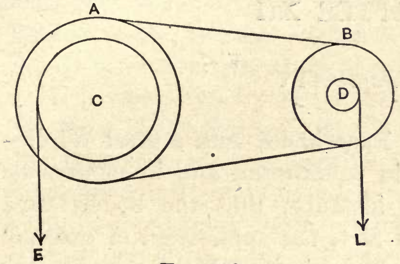

FIG. 168.

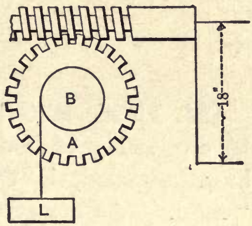

FIG. 169.

14. In an arrangement of gears, as in Fig. 158,

A has 72 teeth,
B " 60 "
C " 32 "

Drum for E, 6 ins. diameter, for L, 4 ins. diameter. Find efficiency of the apparatus and the force required at E to overcome friction when a force of 30 lbs. at E will raise a load 85 lbs.

15. A 24-in. pulley on an engine shaft makes 210 revolutions per minute and drives by belt a 30-in. pulley on a second shaft. This shaft carries a worm which gears with a wheel on a third shaft. If this wheel has 108 teeth, compute speed of third shaft.

16. An engine shaft running 120 revolutions per minute carries a pulley 42 ins. diameter, which drives by belt a pulley 27 ins. diameter on a line shaft. The line shaft carries a second pulley 48 ins. diameter, which connects by belt to a pulley 20 ins. diameter on a counter shaft. Speed of the counter shaft?

What size second pulley must be used on the counter shaft to connect by belt to a dynamo armature having an 8-in. pulley, if speed of the dynamo is to be 1400 revolutions per minute?

CHAPTER XII

ELASTICITY

124. Elastic Material. Experiment with a Steel Wire.—In our study of bodies in equilibrium and in motion we have assumed, in every instance, that the bodies were "rigid." As a matter of fact, the application of external forces always *produces a change in the form* of a body: the amount and character of such change being dependent upon the amount and arrangement of the applied forces, and upon the material of which the body is composed.

As a simple illustration, consider a wire, as AB Fig. 170, attached to a rigid support at A and carrying a scale-pan at B. By adding suitable weights to the scale-pan, we may subject the wire to a pull of any desired amount. To determine the effect, suppose we place the wire in slots in the guides GG to prevent swinging, and then fasten a thread attached to the short arm of a light pointer P, moving freely about an axis at D, to a point on the wire by means of a small spring clip C* clasped about the wire. If the wire stretches, the clip C will be carried downward, the pointer will be rotated about the axis, and the long arm will be moved along the steel scale S by an amount equal to $\dfrac{\text{length of long arm}}{\text{length of short arm}} \times$ distance C moves.

* The ordinary triangular spring wire clips used for fastening together loose sheets of paper have proven perfectly satisfactory for this purpose.

To correct for errors in our experiment arising from the sagging of the support at A, etc., a second pointer should be attached in the same manner to a point on the wire near A. The length of wire under investigation is thus limited to the portion *between the clips* C and C', and the actual stretch produced in this length by any given load at L is measured by the amount that the *lower clip moves more than the upper.* By this arrangement, a wire 8 ft. or more in length may be used conveniently in an ordinary room. With pointers 10 ins. from scale to axis D and 1 in. from D to center of hole where thread from clip is attached, and scales on which the positions of the pointers may be read to $\frac{1}{100}$th inch, the stretch in 8 ft. of wire can be measured to about $\frac{1}{1000}$th inch, or about $\frac{1}{100,000}$th inch in each inch of length. Closer readings should not be attempted.

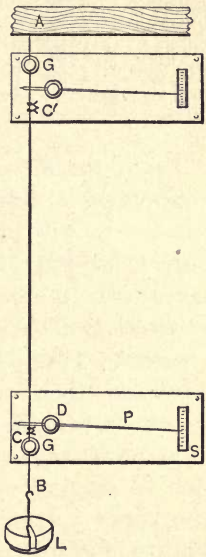

Fig. 170.—Apparatus for Elasticity Test.

In performing our experiment, suppose we first apply a load of 10 or 20 lbs. to remove any kinks from the wire, and then having first set each pointer near the bottom of its scale and noted its position, suppose we add weights in 5-lb. steps to the scale-pan, and read the positions of the pointers after each increase in load. Since we are concerned for the moment only with the effect of a given increase in load, we may disregard the effect of the "straightening load" and assume that we start from zero load.

Readings for a piano steel wire 0.0348 in. diameter and 90 ins. long between clips are shown in the following table:

Added Load. Lbs.	Position of Pointers.		Total Apparent Stretch.	Total Actual Stretch.	Stretch Per Inch (Strain.)	Load in Lbs. Per Sq. Inch (Stress).
	Upper.	Lower.				
0	1.14	0.16				
5	1.19	0.37	0.16	0.016	0.00018	5,300
10	1.21	0.56	0.33	0.033	0.00037	10,500
15	1.23	0.73	0.48	0.048	0.00053	15,900
20	1.24	0.90	0.64	0.064	0.00071	21,200
25	1.25	1.06	0.79	0.079	0.00088	26,500
0	1.23	0.25	0.00	0.000	0.00000	00,000

From these figures, it is apparent that the wire when supporting a scale pan and weights was longer than the same wire with no load; in other words, a *pull caused the wire to stretch.* Also, that the wire continued to stretch more and more as the load increased but *immediately returned to its original length when the stretching load was removed.* This is the characteristic property of *elastic material;* therefore we may say that the *steel wire was elastic* under the conditions present in the experiment. (The permanent change in the positions of the pointers was due to sagging of the support, stretch in the wire above the upper pointer, etc., as was evidenced by the fact that both pointers remained displaced by the same amount.)

DEFINITION.—ELASTICITY *may be defined as the property of bodies by virtue of which they resume their original form when the external forces acting on them are removed.*

125. Stress.—When a load is placed in the scale pan of the apparatus, Fig. 170, a pull equal to the total weight of scale pan and contents is applied to every section across the wire tending to separate the particles of steel along this section from those next above them. This result is opposed by the cohesion between the pairs of particles, and thus the pull is transmitted through the wire to the sup-

port. The *total pull* at a section is *distributed over all the pairs* of particles along that section. If we divide the total pull by the area of section in square inches, we shall have the *pull per square inch* to which the material is subjected. This is called the STRESS.

$$\textit{Stress} = \textit{force per square inch} = \frac{\text{total force}}{\text{area of section}}.$$

Thus when the total pull is 10 lbs., the *tensile stress* in the wire is $10 \times \frac{1}{(0.0348)^2 \times 0.7854} = 10{,}500$ lbs. per sq.in. If the cross section of the wire is not uniform, the greatest stress occurs where the diameter is least.

A stress may be *tensile*, as in the preceding illustration; *compressive*, as in columns and struts supporting a load, in which case the particles are pushed closer together tending to crush the material; or *shear*, in which one portion of material tends to slide past another, as in the relative motion of the blades of a pair of shears. A shear stress is exerted in punching holes in a plate, in bending beams, etc. Shear stress will be considered in more detail in Article 132.

126. Strain.—When stress is applied to a body, the latter undergoes change of form; i.e., is stretched, compressed, bent, twisted, etc. *The fraction of its original size by which the body changes, i.e., its change of form per unit of original size, is called the* STRAIN.

$$\textit{Strain} = \frac{\text{total change in size}}{\text{original size}}.$$

Thus when supporting a weight of 10 lbs., the steel wire originally 90 ins. long was found to have increased in length

0.033 ins. The total stretch was therefore 0.033 ins., and *the strain* $\frac{0.033}{90}$ *=0.00037 ins. per inch.* If a column 10.00 ins. long shortens under compression until only 9.98 ins. long, the change in length is 0.02 in. and the *strain is* $\frac{0.02}{10.00}$ *=0.002 ins. per inch.*

127. Hooke's Law.—If we plot the curve between the last two columns of the preceding table of data, we shall obtain the curve of Fig. 171. This is a straight line passing through the origin, thus showing that for the range of stress to which the wire was subjected, *stretch was directly proportional to load.* This illustrates an important general law, for all elastic material, whether under tensile, compressive, or shear stress. In its general form, this law may be expressed: "*Strain is directly proportional to stress*" (Hooke's Law).*

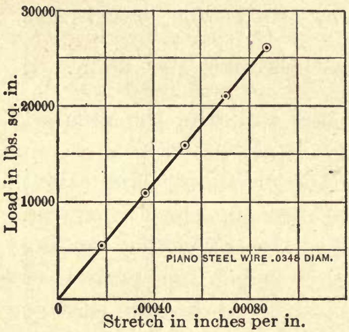

FIG. 171.—Curve showing relation of stretch to load for a piano steel wire.

128. Modulus of Elasticity.—The *modulus of elasticity,* or *Young's modulus,* for a material (sometimes called also its *coefficient of elasticity*) is expressed by the ratio $\frac{\text{stress}}{\text{strain}}$.

* We may now see why all elastic vibrations are *simple harmonic* At the position of extreme displacement, i.e., greatest strain, the returning force is greatest, hence here also the acceleration is a maxmium and toward the center. At the center, stress, and therefore acceleration, is zero.

Modulus of elasticity for tension, $E = \frac{\text{stress}}{\text{strain}} = \frac{\text{lbs. per sq.in.}}{\text{stretch per in.}}$.

For our piano steel wire, modulus of elasticity, $E = \frac{21{,}200}{0.00071}$ $= 30{,}000{,}000$ nearly, which is practically the accepted value for most grades of steel.

The *modulus of elasticity for compression* may be found in a similar way for *short pieces* or for longer ones which are prevented from bending. The piece is subjected to a known load tending to shorten it, and the amount of decrease in length is measured. Stress then equals load $\div$ area of cross section; strain equals amount the piece is shortened $\div$ original length of piece. The modulus of elasticity for compression has practically the same value as that for tension, i.e., compressive strain $= \frac{1}{30{,}000{,}000} \times$ compressive stress.

Long columns, if not supported laterally, yield by bending. In such cases, as also the deflection of beams under transverse load, the stress is not uniform over the section. The computation for stress and for modulus of elasticity for bending is more complicated, and will be considered later.

129. Elastic Limit.—We have seen that for the stress applied in our experiment with the piano steel wire, strain was directly proportional to the stress producing or accompanying it, and that under these conditions the steel was elastic and therefore recovered immediately from strain when the pull was removed. If, however, we had continued our experiment by increasing the load in the scale pan still farther at the same rate, we should soon have reached a stress at which neither of these statements are true. In other words, we should have found, that while for stresses below a certain limiting value, steel is elastic and stretches in proportion to load, *for stresses exceeding*

this limit stretch is more rapid and the material is no longer perfectly elastic but *takes a vermanent "set"* i.e., remains permanently lengthened.

The curves of Fig. 172, which were obtained from tests of an annealed steel wire, will make these statements clearer. This wire was 0.048 in. diameter and 86 ins. long. Stretching loads were applied in the same manner as before but were steadily increased until the wire broke at 109 lbs. actual pull, after a total elongation of about 10 ins. The dotted curves shows the results for loads below 85 lbs., with loads plotted to twenty times the scale.

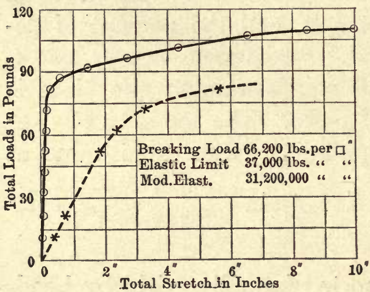

FIG. 172.—Strain diagram for an annealed steel wire.

This curve is a straight line up to a pull of about 45 lbs., or 27,500 lbs. per sq.in., and therefore Hooke's Law applies for this range. As the load increases, however, the curve deviates slightly from a straight line, and at about 60 lbs. pull, or 37,000 lbs. per sq.in., elongation occurs at a perceptibly faster rate than increase of load. This limiting stress measures the *elastic strength* of the material. It also fixes what is known as the *elastic limit* of the wire or the point at which the wire ceases to be perfectly elastic and begins to take set. As will be seen from the curve, the elastic limit is approached gradually and its exact value is often difficult to determine with certainty.

At a load slightly greater than that at the elastic limit (here about 75 lbs. pull), the wire begins to stretch very rapidly. The stress at which this occurs for any material is known as the *yield point.*

The full curve, or as it is commonly termed, the "*strain diagram*" of Fig. 172, therefore furnishes a complete history of the behavior of the specimen of annealed steel wire when subjected to steadily increasing tension. It may be taken as in many ways typical of the behavior of all elastic material under an increasing stress of any character. A portion of the stress-strain curve is a straight line, which indicates the range of stress under which the material is elastic. The elastic limit and yield point of the material are more or less clearly marked, and the rapid strain under higher stress and the total strain when rupture occurs are clearly indicated. As the *safe load* for any particular member of a structure *must always be less than the load at the elastic limit,* the importance to the engineer of a knowledge of the behavior of each kind of material used under the stress to which it is to be subjected is evident. Ample allowance must also be made for accidental overloads.

The ratio $\dfrac{\text{breaking load}}{\text{safe load}}$ for any part of a structure is called its *factor of safety.* Values for the factors of safety in common use, and also for the ultimate strength, modulus of elasticity, etc., of a few common materials will be found in the Appendix. For more complete data the student should consult any good engineering handbook or text on Materials of Construction.

130. Ultimate Strength.—The *ultimate strength* of any material for tension, compression, or shear, is the number of pounds per square inch of tensile, compressive, or shearing force required for rupture. Thus if a wrought-iron bar $\frac{3}{4}$ ins. diameter breaks at an actual pull of 28,700 lbs., its ultimate strength for tension is

$$28{,}700\times\frac{1}{(\frac{3}{4})^2\times 0.7854}=65{,}000 \text{ lbs. per sq.in.}$$

The ultimate strength of the ordinary materials of construction for simple tensile or compressive stress has been pretty definitely determined. Ultimate strengths for shear, and especially for cases in which the material is subjected simultaneously or in alternation to more than one type of stress are not so definitely known. Safe loads for such conditions are fixed largely by practical experience.

131. Conditions Affecting Strength and Elasticity.—The ultimate strength and elastic properties of a specimen depend not only upon the material but also to some extent

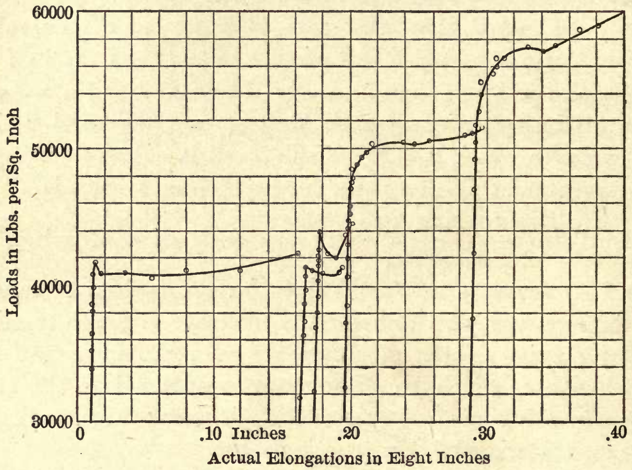

FIG. 173.—Curves showing elevation of elastic limit due to repeated strain.

upon the previous treatment to which the material has been subjected, upon its temperature, etc. Repeated hardening and annealing seems to strengthen steel. Cold-rolled pieces which are finished cold between polished rollers and thus reduced in size (strained) show, on test, greater ultimate strength and a higher elastic limit than

the original material. Specimens taken from the interior of large, solid shafts give usually much lower values for strength and elastic limit than those obtained with specimens taken nearer the circumference, thus showing the effect of difference in working and tempering.

In general, any material loaded repeatedly beyond its elastic limit with periods of rest between, shows on test after each such treatment, that its elastic limit has been raised. Fig. 173 shows the results of such a series of tests performed in the Strength of Materials Laboratories of Pratt Institute upon a Bessemer steel bar 18 ins. long and $\frac{3}{4}$ in. diameter. The loads were applied by an Olsen testing machine, and elongations taken with an extensometer. After each test in which the stress was increased beyond that at the elastic limit, the specimen was allowed to rest for two days before retesting, except for the fifth and last test, which was continued until the specimen broke. Fig. 173 shows the results of the first four tests. The elastic limit was found to be higher in each test except the second.

In Fig. 174 the fifth test and a test of a second fresh specimen cut from the same steel bar are shown together. The repeated stresses beyond the elastic limit had increased the ultimate strength of the material about 15% and had raised the elastic limit, but the ductility of the material was decreased until the elongation was nearly 34% less.

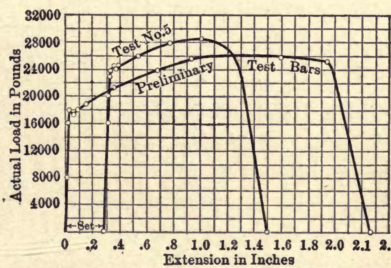

FIG. 174.

It is an interesting question as to how long repeated straining would continue to have the effects here indicated,

and upon this point, owing to the nature of the problem, reliable data is not plentiful.* It is apparent, however, that though at first the material is rendered stronger it is also made more brittle, so that ultimately it breaks with little or no elongation. We may assume, therefore, that every time a specimen is overloaded and given a "set" it is brought just so much nearer its ultimate failure.

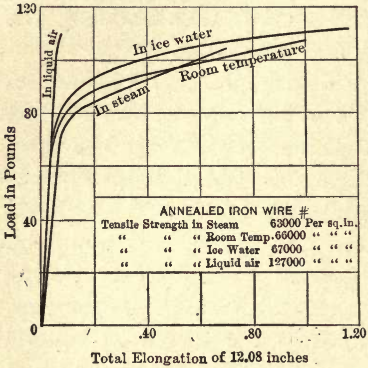

FIG. 175.—Curves showing the effect of temperature on elastic strength of annealed iron wire.

Figs. 175 and 176 show the results of some tests, conducted in the Physics Laboratories of Pratt Institute, of the relative ultimate strength of iron and copper at room temperature (70° F.), at steam temperature (212° F.), in ice water (about 40° F.), and in liquid air (−312° F.). These curves show that both ultimate strength and elastic limit are greatly affected by temperature. The tensile strength of iron was found to be about 95% greater at −312° F. than at 70° F., and that of copper about 50% greater.

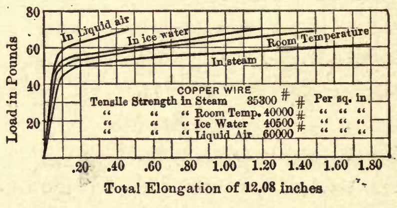

FIG. 176.

* Interesting data of actual tests may be found in pamphlets published by the Bethlehem Steel Co., and in results obtained by Government engineers at the Watertown Arsenal.

These results are shown even more strikingly in the curves of Fig. 177, in which tensile strength is plotted against *absolute temperatures* Fahrenheit. From these curves it appears that the tensile strength of both iron and copper tends to approach a minimum (which for iron is about the temperature of boiling water), after which it again increases. These results are borne out by tests made at the Watertown Arsenal, in which it was found that for wrought iron a minimum tensile strength is reached at a temperature between 200° F. and 300° F. (660°–760° absolute), after which the strength again increases as the temperature rises until about 600° F. when the strength is 20% to 25% above normal. From this temperature the tensile strength decreases rapidly with rise in temperature.

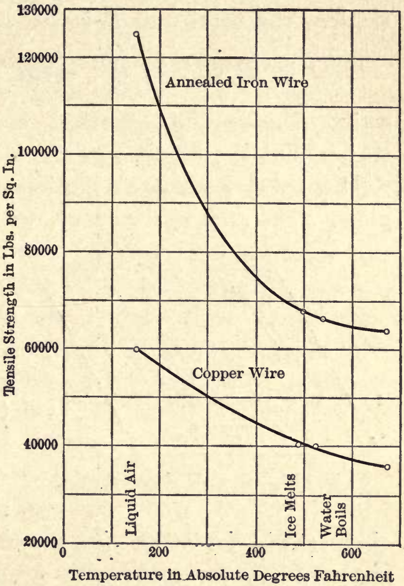

Fig. 177.

132. Simple Shear.—Suppose a rectangular block of elastic material, as *ABCDEFG*, Fig. 178, to be attached to a rigid support over the entire face *EFG*, and suppose a horizontal force *P* to be *distributed uniformly* over the opposite face *ABCD*. Further, assume that the force per square inch over *ABCD* is transmitted uniformly, from

plane to plane, down through the block to EFG. Under these assumed conditions the block will be under a *simple shear stress* tending to slide the particles at any horizontal section over those below, and the block will take the new position $A'B'C'D'EFG$ without bending.* In other words, the block will undergo a *simple shear strain.* If, for illustration, the total pull P is 100 lbs. and the area of surface $ABCD$ is 10 sq.ins., the shear stress is $\frac{100}{10}=10$ lbs. per sq.in. Under the assumed conditions this is uniform throughout the block.

The total horizontal motion will evidently be zero at plane EFG and will increase with the vertical height to a maximum at the plane $ABCD$. Thus, if $AE=2ME$, the motion AA' will equal $2\times MM'$. *The horizontal movement at a distance one inch above the fixed plane is defined as the shear strain.* Thus if $EM=1$ in. and $MM'=0.0005$ in., the *shear strain is 0.0005 in. per inch.* At a plane 1.4 ins. above EFG, the movement would be $1.4\times0.0005=0.0007$ ins.

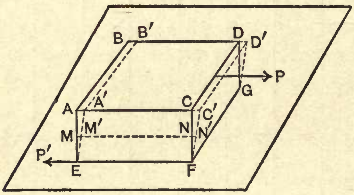

FIG. 178.—Simple shear strain.

In this case, as for tension, shear strain will be proportional to shear stress until the elastic limit of the material is reached, and upon removal of the stress the block will return to its rectangular form. Stresses greater

* The conditions here assumed can only be rigidly true when other forces are applied to the block to prevent bending. If, however, the thickness AE of the block is small as compared to its length AC, our assumed conditions would be approximately true for the middle portion of the block.

than the elastic limit will give a permanent set to the material.

The ratio $\frac{\text{shear stress}}{\text{shear strain}}$ is called the *modulus of rigidity.* The modulus of rigidity of steel is about 13,000,000.

The shear stress required for rupture measures the ultimate strength of a material for shear. A common illustration of rupture under a shear stress is furnished in punching rivet holes in a plate. The area of the curved surface of the hole (circumference of hole×thickness of plate). multiplied by the strength of the material of the plate for shear represents the force with which the punch must be pressed against the plate.

133. Torsion.—If we mark parallel straight lines from end to end upon the surface of a flexible rubber rod and then clamp one end fast and apply to the other end a couple which tends to rotate this end about the axis of the rod, we find that the twist is communicated along the entire rod to the end which is fast in such a manner that the straight lines become helices, about the axis of the rod. The actual displacement of a point in any line from its original position varies directly with its distance from the fixed end, being zero at this end and a maximum at the free end where the torque is applied. This is the general phenomenon known as *twisting* or *torsion.* A little consideration will make it clear that the *strain* imparted to the material of the rod *is a shear strain.* The fibers have been displaced through an angle with the axis of the rod, and the particles at every section have been rotated through an arc the center of which lies at the center of the section. The outer particles at a section as P, Fig. 179, have moved

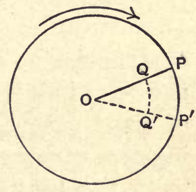

FIG. 179.

to the new position P' a particle nearer the axis, as Q has moved to the new position Q', while particles at the center O have not been rotated at all. The *shear strain* at a section is greater and greater, as the material lies farther from the axis O; it is also greater the farther the section is taken from the fixed end of the rod.

134. Laws of Torsion.—A simple laboratory apparatus for the study of the laws of torsion is shown in Fig. 180. A clamp, which may be adjusted to rods of different diam-

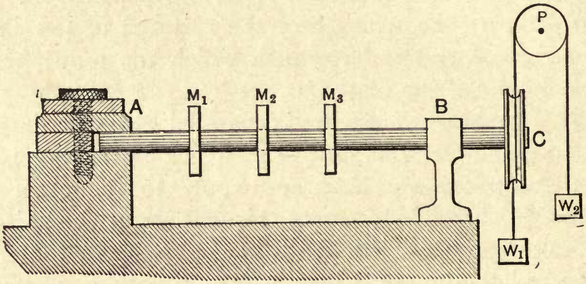

FIG. 180.—Apparatus for studying laws of torsion.

eter, holds the end A of the rod to be twisted; the other end rests in a loose bearing B. Torque is applied by means of cords passed around the grooved wheel C and supporting the equal weights W_1 and W_2. In this way pressure upon the bearing B is avoided. Adjustable mirrors, M_1, M_2, M_3, etc., are attached to the rod by sliding spring sleeves, at the equal intervals AM_1, M_1M_2, M_2M_3, etc., and the amount of twist for each length is determined by viewing in these mirrors, by means of a telescope, the image of a vertical scale placed at relatively so great a distance that it is practically equally distant from each mirror. The twist may be taken as proportional to the

reading in scale divisions for each mirror, or the actual angle of twist may be found from the relation:

$$\frac{\text{Change in scale reading}}{\text{Distance scale to mirror} \times 2} = \text{tan. angle of twist.}$$

With this apparatus it may be shown experimentally that:

1. If a twisting moment T produces an angle of twist ϕ, a moment of $2T$ will produce a twist of 2ϕ, etc. Or, *angle of twist is proportional to twisting moment.*

2. If distance $AM_1 = M_1M_2 = M_2M_3$, etc., and twist at M_1 is 10 scale divisions, twist at M_2 will be 20 divisions, at M_3 30 divisions, etc. Or, *angle of twist for the same torque is proportional to length of rod.*

3. For round rods or shafts of different diameters, *the angle of twist for the same torque varies inversely as the fourth power of the diameter.*

4. For rods of the same size but different materials, the twist for the same torque varies as $\frac{1}{\text{modulus of rigidity}}$.*

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* (*a*) *Modulus of Rigidity by Torsion.*—It may be shown that the torque T required to twist a rod of length l and diameter D through an angle of ϕ degrees, is expressed by the equation $T = NI\alpha$, where N is the modulus of rigidity for the material, I the moment of inertia of the section, α the *twist per unit of length expressed in radians.* α evidently equals $\frac{2\pi\phi}{360l}$ and for round rods $I = \frac{\pi D^4}{64}$. Therefore N may be computed by the equation,

$$N = \frac{T}{\frac{\pi D^4}{64} \times \frac{2\pi\phi}{360l}}.$$

(Dimensions should be in inches, T in pound-inches.)

(*b*) *Modulus of Rigidity of a wire by Torsion Pendulum.*—The modulus of rigidity of a wire may be found by using it as the suspension

5. Twisting moments required for rupture, other things being the same, are proportional to the cube of the diameter of the rods.

PROBLEMS

1. A cast-iron bar which is to be subjected to a tension of 34,000 lbs. is to be designed so that the unit stress shall be 2500 lbs. per sq.in. What should be the sectional area in square inches? If the bar is round what should be its diameter?

2. A round rod of wrought iron $2\frac{1}{2}$ inches in diameter, ruptures under a tension of 271,000 lbs. What is its ultimate strength?

3. A steel eye-bar 30 ft. long is $1\frac{1}{2}\times 6$ ins. in size. How much does it elongate under a pull of 90,000 lbs.?

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for a heavy disk of known moment of inertia about an axis through its center. The wire should be attached at the center of the disk. Then if this pendulum be set vibrating, without swinging, about a vertical axis, and its period t determined, modulus of rigidity may be found from the relation:

$$\text{Modulus of rigidity of wire} = \frac{\pi I p l}{g^2 t^2 r^4},$$

where Ip is the polar moment of inertia of the disk, l the length of the wire in feet, r its radius in feet, t the period of vibration in seconds.

If the modulus of rigidity of the suspension wire is known, it should be noted that this equation also enables us to determine the moment of inertia of bodies which may be suspended in place of the disk about an axis which is the axis of the suspension wire continued.

Or, if the modulus of rigidity of the wire is unknown, we may still find the moment of inertia of a body by use of a second body whose moment of inertia is known or readily computed, as follows: Suspend one body I_1 and determine the period t_1. Then add the second, I_2, and determine the period t_2 for the two combined. Then,

$$\frac{I_1}{I_1+I_2} = \frac{t_1^2}{t_2^2},$$

from which either I_1 or I_2 may be found if the other is known.

4. A bar 1 in. square and 2 ins. long elongates 0.0004 in. under a tension of 5000 lbs. Compute the coefficient of elasticity.

5. A wire 1 mm. diameter breaks under pull of 125 lbs. Under what load will a wire of the same material and with diameter 1.65 mm. break?

6. A steel column 6 sq.ins. in section and 8 ins. long bears a load of 75,000 lbs. How much will it be compressed?

7. What must be the sectional area of the legs of the shears, Fig. 91, in order that, assuming no bending, the stress may not exceed 10,000 lbs. per sq.in.? Assume a load of 100 tons, a 20° angle between legs, a 45° angle between tie and horizontal, a 15° angle between plane of legs and vertical, and that the hoisting rope leaves the upper block parallel to the tie.

8. The tie in the derrick of Fig. 15 has a tensile strength of 90,000 lbs. per sq.in. It has 3 strands each $\frac{7}{8}$ in. in diameter. What load at L will be required to break it? Assume

Angle $DEC = 100°$,
" $EDC = 40°$,
" $CDL = 40°$,

and disregard the weight of DC.

9. A plate $\frac{9}{16}$ ins. thick has a shearing strength of 60,000 lbs. per sq.in. What force will be required to drive a punch $\frac{5}{8}$ in. diameter through the plate?

10. The tension members of the truss, Fig. 96, consist of flat. wrought-iron strips, $\frac{3}{4}$ in. thick. How wide must each be in order that the stress may not exceed 7500 lbs. per sq. in.? What will be the strain for this load?

11. A tie rod 28 ft. long, 2 sq.ins. section, carries a load of 30,000 lbs. It is stretched $\frac{3}{16}$ in. Find stress, strain, and modulus of elasticity.

12. What will be the deflection due to shear in a beam 8 ins. long and 2 sq.ins. section, if it is fixed at one end and a load of 30,000 lbs. is placed at the other end? Assume a modulus of rigidity of 13,000,000.

135. Bending.—Fig. 181 shows a convenient laboratory device for illustrating the general laws of bending. The beam to be bent, which may be either a wooden or metal rod of any desired section, rests upon adjustable knife edges supported rigidly upon a lathe bed. Bending loads are applied by scale pan and weights suspended from the middle of the beam, and the resulting deflections are measured by determining with a micrometer screw the distance

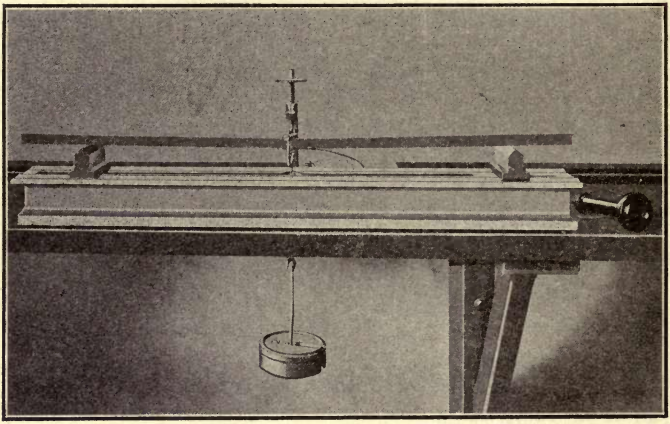


FIG. 181.—Apparatus for illustrating laws of bending.

from a fixed level to the surface of the beam at the middle. Where more precise measurements are desired, the micrometer screw may be replaced by an optical lever carried on the arm which here supports the micrometer screw, the motion of which is measured in terms of the divisions of a scale placed a known distance from the mirror of the lever.

Results obtained in the laboratory with this apparatus are shown by Fig. 182. It will be seen from these curves, that the *deflection is proportional to load* when the elastic limit of the beam is not exceeded. For this range of stresses

also, the beam returns to its original position when the bending load is removed. If loaded beyond the elastic limit, the beam takes a permanent "set."

136. Neutral Axis.—Nothing is shown in the preceding experiment as to the character or distribution of the stress to which the beam is subjected. If, however, we examine a beam when bent, we may form definite conclusions upon these points. Fig. 183 shows such a beam with the deflection greatly exaggerated for the purposes of illustration. A uniformly distributed load, in place of a simple, central load, is assumed for the same reason.

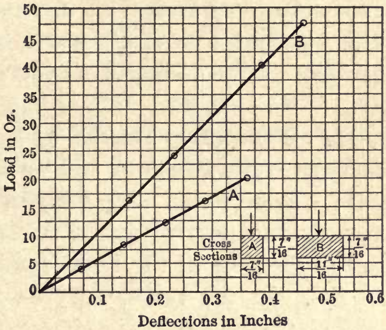


FIG. 182.—Curves showing relation of DEFLECTION TO LOAD with different WIDTHS.

It is apparent from this diagram that the edge AB of the beam is *shortened* by the bending, and that the edge CD is *lengthened*. The upper portion of the beam must therefore be under *compressive stress*, and the lower portion under *tensile stress*. At some place between AB and CD, as MN, there must be a line which, while curved because of the load, remains of the same length as when the beam was straight. A surface may therefore be imagined

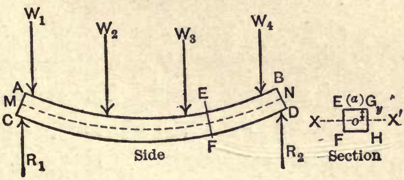


FIG. 183.—Diagram of a loaded beam, bending exaggerated.

across the beam through MN, above which the fibers are under compression and below which they are under tension, but in which there is neither compression nor tension. This surface is known as the *neutral surface* for the beam. The straight line XX', which shows the position of this surface across a section of the beam, is called the *neutral axis of the section.*

137. Maximum Tensile and Compressive Stress for a Section.—The fibers of the beam are shortened most at AB; therefore at any section the compressive stress is zero at the neutral axis XX' and increases in proportion to the distance of the fiber from XX' to a maximum stress at EG. In the same way the tensile stress on the other side of the neutral axis is zero at XX' and increases with the distance to a maximum at FH. If a beam fails through compression, it will be most apt to yield first at the outer fibers EG, where the stress is greatest; and if it fails because the tension is too great, it will be most apt to yield at FH, where the tensile stress is maximum. In computing the safe load for a beam, therefore, it is necessary only to consider the stress at the outer fibers for any section.

138. Shear in Beams.—In addition to the tensile and compressive stresses just considered, there is usually a shear stress at each section of a loaded beam. The model shown in Fig. 184 may be used to illustrate this. A portion $CDEF$ has been removed from the beam, and to supply the tensile and compressive forces a cord has been inserted at CD and a rod capable of resisting compression only has been placed at

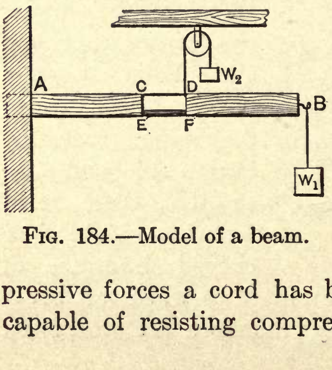


FIG. 184.—Model of a beam.

EF. If the end *A* is held rigidly, and a bending load *W* applied at *B*, it is found that an additional force, the vertical pull W_2, must be introduced at the section *DF* to support the outer end of the beam. This represents the *shearing force* exerted on the section *DF* by the material at the left. It may be shown experimentally also that $W_2 = W_1 +$ weight of end *DB* of beam.

The presence of a shearing force exerted by the remaining portion is apparent also when we consider the static conditions for a portion of a beam. Thus suppose we represent the portion of the beam in Fig. 183 to the left of the section *EF* as a "free body"; the forces acting (see Fig. 185) are the reaction R_1 of the support, the loads W_1, W_2, W_3, and the weight of the beam, and the molecular forces at the section coming from the material at the right. These will consist of a push *S* distributed over a portion of the section and a pull *T* distributed over the remainder. The conditions for equilibrium are:

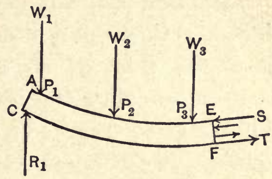


FIG. 185.—Force on a part of a loaded beam.

(1) Sum $X = 0$;

(2) " $Y = 0$;

(3) " (moments) $= 0$.

We know, from the assumption of a *symmetrical beam symmetrically loaded*, Fig. 183, that $R_1 = R_2$, and therefore, that R_1 *is less than* the sum of W_1, W_2, W_3, and the weight of portion *AE* of the beam. To satisfy condition (2), therefore, *the molecular forces at the section EF must have an upward, vertical resultant Q of such value that:*

$$Q = W_1 + W_2 + W_3 + \text{weight of beam } AE - R_1;$$

Q therefore represents the shearing force at the section EF. It is evident from a little consideration that Q is greatest at each end of the beam where it is equal to $\frac{1}{2}$ (total load + weight of beam), and zero at the middle, since $R_1 = R_2 =$ sum of load and weight of beam either side of the section.

The student should assume actual loads and dimensions and compute the value of the shearing force at several sections along a beam loaded as above, and also for other common types of support and loading shown in Fig. 186.

139. Bending Moment.—The sum of the moments of all forces acting upon the portion of a beam between an end and any selected section, about the neutral axis of the section, is called the *bending moment for that section.* Thus referring to Fig. 185, the bending moment M for the section at EF is expressed by the equation:

$$M = W_1 \times P_1E + W_2 \times P_2E + W_3 \times P_3E + \text{weight of } AEFC \times \tfrac{1}{2}AE - R_1 \times AE.$$

The *maximum bending moment* for the beam of Fig. 183 is evidently for a section at the middle, where if l is the length of the beam,

$$M = R_1 \times \tfrac{1}{2}l - \tfrac{1}{2}(\text{total load} + \text{weight of beam}) \times \tfrac{1}{4}l.$$

Or, since $R_1 = R_2 = \frac{1}{2}(\text{total load} + \text{weight of beam})$,

$$M = \tfrac{1}{8}(\text{load} + \text{weight of beam})l.$$

The student should assume actual loads and dimensions and compute the bending moments at several sections along a beam with loads and supports as in Fig. 183, also for the common types shown in the diagrams, Fig. 186. The relative bending moments of different sections of a beam may be shown most conveniently by diagrams, as in

Fig. 186, in which the bending moments at successive sections are represented by the vertical distances to the dotted line.

I. Beam fixed at one end, load W at other. Maximum bending moment at $A = Wl$. Shearing force is equal and opposite to W everywhere.

II. Beam as in I, load W uniformly distributed. Maximum bending moment at $A = W \times \frac{1}{2}l$. Shearing force is maximum at A, less toward B.

III. Beam supported at both ends, load W in middle. Maximum bending moment at middle, $= \frac{1}{2}W \times \frac{1}{2}l$. Shearing force is $\frac{1}{2}W$, A to middle, $-\frac{1}{2}W$, B to middle.

IV. Beam as in III, load W uniformly distributed. Maximum bending moment at middle $= \frac{1}{8}Wl$. Shearing force is $\frac{1}{2}W$ at A, $-\frac{1}{2}W$ at B, zero at middle.

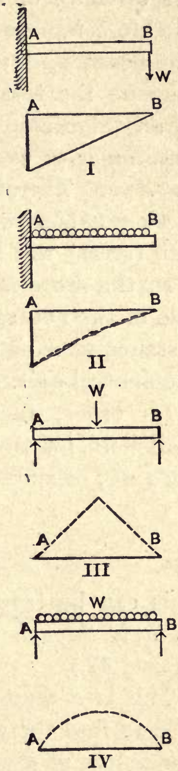


FIG. 186.—Diagrams of bending moment.

140. Neutral Axis always Through the Center of Gravity of the Section.—It is apparent from Fig. 185 that all forces acting upon the "free body" are vertical except the molecular forces at the section EF. To satisfy the con-

dition, Sum $X=0$, therefore, *the resultant of the horizontal components of all these molecular forces must be zero.*

The compressive stress at any point is proportional to the distance of the point from the neutral axis: we may therefore represent it by Ky, where K is some constant for the beam and y is the distance to neutral axis. If we imagine the total area under compression to be made up of an infinite number of areas a, so small that the compression over each is uniform, the force on each area will be Kya. Therefore the total compressive force for the section (which will be equal to the sum of all these separate forces) will be Sum Kya.

In the same manner the total tensile force for the other side of the neutral axis will be Sum $Ky'a$, where y' is the distance to each area measured in the other direction from the neutral axis.

In order, therefore, that the condition Sum $X=0$ shall hold for the section, we must have Sum Kya − Sum $Ky'a=0$, or since K has the same value in each,

$$\text{Sum } ya = \text{Sum } y'a.$$

This can be true only when y and y' are measured from an *axis through the center of gravity of the section.* (See Article 37.)

This is a reason why it is important that we should be able to find the center of gravity of a surface. By finding the center of gravity of the section of the beam we determine the position of the neutral axis.

141. General Equation for Stress.—The application of the third condition of equilibrium, Sum (moments) $=0$, to the free body of Fig. 185 leads us directly to a fundamental equation for the stress in beams. By this condition, referring to the figure, we see that the combined

moments of the loads W_1, W_2, and W_3, the reaction R_1 of the support, and the molecular forces at the section, about the neutral axis of the section, must be zero. In other words, *the molecular forces must supply a moment about the neutral axis which shall balance any difference in the moments of* W_1, W_2, W_3, *weight of beam, and* R_1 *about the same axis.*

But by Article 139, $W_1 \times P_1E + W_2 \times P_2E + W_3 \times P_3E +$ weight of portion AE of beam $\times \frac{1}{2}AE - R_1 \times AE =$ *bending moment M* for the section AE. Therefore the *moments of the molecular forces at a section about the neutral axis equals the bending moment for the section.*

By the preceding article the force on any area a is Kya. The moment of this force about the neutral axis is *force×arm*, or $Kya \times y = Ky^2a$. The total moment of all the forces is the sum of the moments of the separate forces or Sum Ky^2a. Hence Sum $Ky^2a = M$.

But by definition (Article 88) Sum (y^2a) for any area = moment of inertia I for that area about the axis from which y is measured.

Therefore, $KI = M$, and $K = \frac{M}{I}$. Substituting this value for K in our expression for stress, or Ky, we have the general equation:

$$\text{Stress} = \frac{M}{I} \times y,$$

or, *The stress at any fiber for any given section of a beam equals* $\dfrac{\textit{bending moment}}{\textit{moment of inertia of section}} \times$ *distance of fiber from neutral axis.*

This may be regarded as the fundamental equation for beams. Naturally the computation would be made only

for the *maximum stress* anywhere in the beam. This will be at the outer fiber of the section at which the bending moment is maximum. Data for the moments of inertia of various sections may be found in engineering handbooks.

Example 1.— A beam 10 ft. long, $4 \times 10''$ in section, resting on edge, is supported at the ends and bears a distributed load of 200 lbs. per foot. What will be the maximum stress and the shearing force?

Maximum bending moment is at the middle, and is

$$\tfrac{1}{8}wl = \tfrac{1}{8} \times 200 \times 10 \times 120 = 30{,}000 \text{ in.-lbs.}$$

$$I = \frac{bd^3}{12} = \frac{4 \times 10 \times 10 \times 10}{12} = \frac{1000}{3}.$$

Therefore,

$$\text{Maximum stress} = \frac{M}{I} \times 5 = \frac{30{,}000}{\frac{1000}{3}} \times 5 = 450 \text{ lbs. per sq.in.}$$

Maximum shearing force is at the ends, and is

$$\tfrac{1}{2}W = \frac{200 \times 10}{2} = 1000 \text{ lbs.}$$

Example 2.—A round steel rod 18 ins. long in the clear, and $1\frac{1}{2}$ ins. diameter, is fixed at one end. What load may be suspended from the other end if the maximum allowable stress is 5000 lbs. per sq.in.?

Maximum bending moment is at end which is fixed and is $W \times 18$ in.-lbs.

$$I = \frac{\pi D^4}{64} = \frac{891}{3584}.$$

Therefore,

$$5000 = \frac{\frac{W \times 18}{891}}{3584} \times \tfrac{3}{4}, \text{ from which } W = 92.1 \text{ lbs.}$$

142. Modulus of Elasticity by Bending.—The modulus of elasticity of a material may be determined experimentally from measurements of the deflection of a strip or rod under a load which gives a tensile and compressive stress less than that at the elastic limit. If W is the load, l the length of the beam in inches, d the deflection in inches due to W, I the moment of inertia of the section, it may be shown that modulus of elasticity $E = \frac{wl^3}{3dI}$ for a beam fixed at one end, with load w at the other.

For a beam supported at both ends with load w at the middle, the corresponding equation is:

$$E = \frac{wl^3}{48dI}.$$

143. Effect of Dimensions of a Beam upon its Stiffness.—It may be shown that with other conditions constant, the resistance of a beam to bending varies directly with the breadth, inversely with the cube of the length, and directly as the cube of the depth; or

$$\mathit{Stiffness} \propto \frac{bd^3}{l^3}.$$

An experimental study of these laws, with the apparatus shown in Fig. 181, forms an instructive laboratory exercise. The beams should be straight grained wooden rods or metal strips of rectangular section, carefully dressed to

size, and the loads used should be well below those at the elastic limit. Fig. 182 shows the results of a test of the effect of increasing the breadth of the beam. Two rods of the same material, and of the same length and depth, were used, the breadth of one being twice that of the other. The breadths were therefore in the ratio $\frac{1}{2}$. The deflections for the same load were: For 10 ozs., $\frac{.175}{.090}$, for 20 ozs., $\frac{.363}{.187}$.

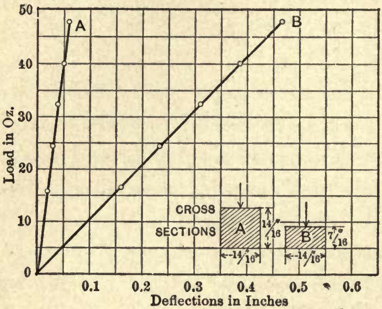


FIG. 187.—Curves showing relation of deflectlon to depth. Material of beam-pine, length 40 inches.

These are very nearly in the ratio $\frac{2}{1}$, showing that the beam of twice the width was bent only $\frac{1}{2}$ as much, or in other words, was *twice as stiff*.

Fig. 187 shows results of a similar test for the effect of depth on stiffness. The depths were in the ratio $\frac{2}{1}$; the deflections for the same load at the middle of the beam were for 20 ozs., $\frac{.025}{.195}$, for 40 ozs., $\frac{.050}{.390}$. These ratios are both approximately $\frac{1}{8}$ or $\frac{1}{(2)^3}$, showing that if the depth is doubled the beam is $(2)^3$ or 8 times as stiff.

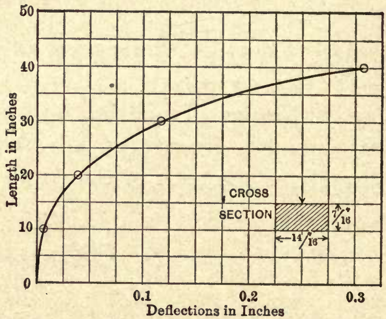


FIG. 188.—Curve showing relation of deflection to length. Material of beam-pine.

Tests of deflections of different lengths of the same beam with the same load are shown in the curve of Fig. 188. The load was kept central and the supports adjusted until successively 40, 30, 20, and 10 ins. apart. The lengths of beam were therefore in the ratios $\frac{4}{1}$, $\frac{3}{1}$, and $\frac{2}{1}$; the corresponding deflections were $\frac{.310}{.005}$, $\frac{.119}{.005}$, and $\frac{.038}{.005}$, or approximately $\frac{64}{1}$, $\frac{27}{1}$, and $\frac{8}{1}$. These are $\frac{(4)^3}{1}$, $\frac{(3)^3}{1}$, $\frac{(2)^3}{1}$, showing that bending increased (and therefore stiffness *diminished*) in proportion to the length cubed.

Very suggestive results may be obtained also with beams other than rectangular in section. Here it should be remembered that the sections being *similar*, *breadths* refer to the relative *lateral dimensions* of the beams, *depths* to the relative *vertical* dimensions.

144. Effect of Dimensions and Manner of Loading upon Strength of a Beam.—It has been shown that while the stiffness of a beam increases as the depth cubed, the *strength increases only as the depth squared.* The strength is also proportional to the breadth of the beam, and inversely proportional to its length. Or,

$$\text{Strength} \propto \frac{bd^2}{l}.$$

Since we may expect rupture to occur when the bending moment at any section of a beam exceeds a certain limiting value, the diagrams of bending moment, Fig. 186, show some interesting relations of the maximum load which a given beam will support to the manner in which this load is applied. Thus a beam of given size and length which will support a maximum load w when fixed at one end and

loaded as in diagram I, may be expected to carry a load $2w$ distributed as in II, $4w$ if both ends are supported and the load concentrated at the middle of the beam as in diagram III, and $8w$ if supported at both ends with the load uniformly distributed as in diagram IV.

145. Shapes of Beams and Columns.—Short columns, or columns prevented from bending, may be expected to yield only at a stress equal to the ultimate compressive strength of the material. Long columns not supported laterally yield by bending.* It is desirable therefore to shape the section of posts and struts to offer the greatest possible resistance to bending. This is accomplished, as is seen from the laws for stiffness just stated, when the material is distributed as widely as possible about the axis of the column.

In beams the top must resist compressive stress and the bottom tensile stress. The bottom flanges may therefore be thin plates as a pull does not produce bending, but the

---

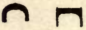
* *Strength of Struts.*—The breaking load F for a short strut or one prevented from bending is expressed by the equation $F=fA$, where f is ultimate compressive strength of the material, A is area of section in square inches.

Various formulæ are in use for struts which yield by bending, based upon the shape and manner of holding at the ends. For a strut with what may be called "free" ends, the load F, in line with the axis of the piece which will produce bending is,

$$F=\frac{EI\pi^2}{4l^2},$$

where E is Young's modulus, I the moment of inertia of a section about the line through its center of gravity, l one-half the length of the strut. For this, and similar formula for other types, the student is referred to advanced treatises in Mechanics.

Where the value of F, obtained by such formulæ, is greater than that resulting from use of the formula $F=fA$, the lower value must be taken as the strut will yield to compression.

top must be in the form of a hollow tube or shaped thus  to resist the tendency to bend under compression.

Cast iron has a compressive strength nearly $4\frac{1}{2}$ times as great as its tensile strength. The section of cast iron girders is therefore usually made so that the area under tension is approximately four and a half times as large as that under compression. Fig. 189(a) shows some common sections for cast-iron girders.

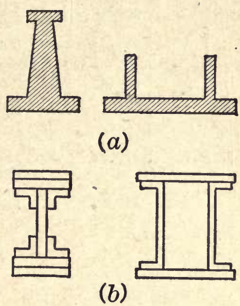


FIG. 189.—Sections of girder.

Steel and wrought iron have practically the same ultimate strength for both tension and compression. Symmetrical and economical "built-up" sections similar to those of Fig. 189(b) are therefore common.

In general the function of the flanges is to resist bending moment, that of the web to resist the shearing force. Any web strong enough to hold the flanges in position and to permit the girder to be handled is usually amply sufficient to withstand the shearing force.

In beams of uniform section, only the maximum bending moment need be considered. The section must be designed to safely sustain this. Girders of "equal strength" may be designed in which the section varies according to variation in bending moment.

146. Stress for Hooks, Brackets, etc.—In the case of hooks, brackets, etc., the applied load acts in a line which does not coincide with the axis of the piece. The material is therefore subjected to both a tensile (or compressive) force and a bending moment. For example, in a hook, as Fig. 190, we may consider the stress at the section AB as

due to the bending moment $W \times CG$, where G is an axis perpendicular to the plane of the paper through the center of gravity of the section, plus a tensile stress, spread uniformly over the section, equal to $\dfrac{W}{\text{area of section}}$. The stress at any fiber in the section is computed as for beams.

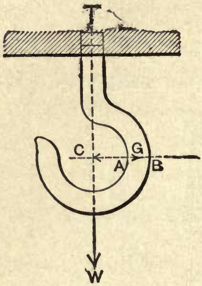


Fig. 190.

If the distance CG is large, as in the case of many brackets, stress due to bending moment alone is important.

PROBLEMS

1. A beam AB, 16 ft. long, weighing 200 lbs. per ft., is supported at both ends. It bears loads as follows: 5 tons 4 ft. from end A, 10 tons at middle, 6 tons 5 ft. from end B. Compute bending moments due to loads and weight of beam combined for sections 2 ft., 4 ft., 8 ft., and 10 ft. from A.

2. In a beam as diagram II, Fig. 186, beam is 6 ft. long and load 500 lbs. per foot. Compute maximum bending moment and shearing force; also bending moments 2 and 4 ft. from fixed end.

3. The top and bottom flanges of a girder are each $6 \times \frac{5}{8}''$. Web is $\frac{5}{8}''$ thick and depth of beam over all is 10 ins. What bending moment will be required to give a maximum tensile stress of 10,000 lbs. per sq.in.?

4. What is the greatest stress in a bar $\frac{7}{8}$ in. diameter, 6 ins. long, when it is fixed at one end and 50 lbs. are suspended from the other end?

5. What will be the greatest stress in a bar 2 sq.ins. in sectional area for a bending moment of 20,000 in.-lbs., if (*a*) round? (*b*) square?

6. A rectangular bar 5 ft. long is supported at the ends. It is 2 ins. wide and 4 ins. deep. What load at the middle will be required to give a maximum tensile stress of 10,000 lbs. per sq.in.?

7. A beam 20 ins. long, 2 ins. deep, and 1 in. wide is planed down so that its depth is $1\frac{1}{2}$ ins. and breadth $\frac{3}{4}$ in. How is its stiffness affected? Increased or decreased, and by how much?

If it were made 5 ins. shorter what would be its stiffness in terms of original stiffness?

8. In a hook, as Fig. 190, the section at *AB* is circular $\frac{3}{4}$ in. diameter. *CG* is 1 in., *W* is 400 lbs. What will be the maximum stress at the section?

CHAPTER XIII

MECHANICS OF FLUIDS

147. Fluids.—Matter may exist in the form of solids, as iron, wood, ice, etc., or in the form of fluids, as water, oil, steam, oxygen, air, etc. Fluids may be either liquids or gases.

The distinguishing property of a solid is that it has a definite shape and volume which cannot be changed without the application of an external force. Thus a piece of iron has a particular shape and an unchanging volume wherever it is placed. A fluid, on the other hand, has no definite shape, but moulds itself to the shape of the vessel that contains it. Thus we may have a given quantity of water or of air in a vessel and may transfer it to another vessel to whose shape it will conform itself. Also a solid cannot be stirred or cut without the application of considerable force, but a fluid offers very little resistance to the disturbing force. These examples show that a fluid is composed of particles that are *mobile*, i.e., they move freely among each other and can easily be separated.

Furthermore, if we take a straight cylindrical tube open at one end and closed at the other, and fit it into a smooth rod of some solid substance such as iron or wood, and press the end of the rod in the direction of its length, the pressure applied is transmitted to the closed end of the tube without producing any effect on the curved surface; but if the tube contains a fluid, either a liquid or a gas, and

pressure be applied to it by means of a tight piston fitting into the tube, pressure will be felt not only on the base of the vessel but also at all points of the curved surface. Thus fluid bodies are those that cannot sustain a longitudinal pressure, however small, without being supported by a lateral pressure also.

148. Distinction between Liquids and Gases.—The difference between a liquid and a gas is that liquids have a definite *volume* but no definite shape, while gases have *neither* a definite volume nor a definite shape. Thus water is, like all true liquids, practically incompressible, but will adapt itself to the shape of whatever vessel it is put into. But gas, like air or illuminating gas, has no definite shape and may also be expanded by the lessening of the pressure upon it or compressed to any extent. These properties of solids, liquids, and gases may also be expressed as follows:

> SOLIDS have elasticity of *both* shape and volume.
>
> LIQUIDS have no elasticity of shape, but perfect elasticity of *volume*.
>
> GASES have elasticity of *neither* shape nor volume.

It should be noticed that the distinctions above are of a general nature only; there are some substances that exist in an intermediate state, like coal tar, thick glue, etc. Also there are vapors which can be made to pass by imperceptible degrees from the gaseous to the liquid state. There are no strict lines of division between the three states of matter. The same substance may, under differing conditions of temperature, pressure, etc., exist in any one of the three states, as steam, water, and ice. Iron which is, as we know it, either a solid or a liquid, exists in the sun only as a gas.

149. Density.—Iron is *more dense* than wood and water is *more dense* than air. By these statements we mean that

a given volume of one substance *has more weight* (or mass) than the *same volume* of the other.

When we wish to state the density of a substance we give the *weight of a unit volume of it.* Thus the density of iron is *7.8 gm. per cubic centimeter;* the density of water is *62.5 lbs. per cubic foot,* and of glass is *1.5 oz. per cubic inch.* Expressed algebraically this definition is as follows:

$$\text{Density} = \frac{\text{Weight}}{\text{Volume}}; \text{ therefore } \textit{Weight} = \textit{Volume} \times \textit{Density}.$$

A knowledge of the density of the common materials of construction is very valuable to designers and builders because it enables them to compute the approximate weight of any particular part of a machine or structure. The volume of a part determined from the scale drawing $\times$ the density $=$ weight of finished part. The densities of a few of the more common materials of construction are given in the Appendix.

150. Specific Gravity.—By the SPECIFIC GRAVITY of a substance we mean its *relative density as compared to water;* or in other words, *how many times heavier a particular body is than an equal volume of water.** Thus we say the *specific gravity* of brass is 8.4; meaning that brass is 8.4 times as heavy as water. A cubic foot of brass, therefore, weighs approximately $8.4 \times 62.5 = 525$ lbs.

$$\text{Specific gravitv} = \frac{\text{Weight of the substance}}{\text{weight of an equal volume of water}}.$$

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* Strictly, the comparison should be to an equal volume of *pure water at 4° C.*, the temperature at which water is most dense. The density of water changes so slightly with change of temperature, however, that for practical purposes the comparison may be made at any convenient temperature.

If we use the metric units, the *density* of a body and its *specific gravity* are expressed by the same number. Thus the density of lead is 11.35 gm. per cc., and its specific gravity is 11.35. But if we are using the English units, we should say that the specific gravity of lead is 11.35 and therefore its density is $11.35 \times 62.5 = 709$ lbs. per cubic foot.

PROBLEMS

(Consult tables for required density or specific gravity.)

1. Find the weight of a bar of copper whose cross-section is 14 sq.cm. and length 45 cm.

2. Find the size of a round iron shot to weigh 16 lbs.

3. Find the weight of a column of mercury 30 ins. high and having a cross-section of $\frac{1}{2}$ sq.in.

4. What is the weight of the air in a room $20 \times 30 \times 12$ ft.?

5. The specific gravity of sea water is 1.03. How many cubic feet of water are displaced by a 400-ton ship?

6. Find the weight of a gallon of sulphuric acid, specific gravity 1.84.

7. Four cubic feet of cork weigh 60 lbs. What is specific gravity of the cork?

8. What will be the weight of 40 ft. of lead pipe, inside diameter 2 ins., outside diameter $2\frac{5}{8}$ ins.?

9. What weight of sheet copper $\frac{1}{16}$ in. thick will be required to line a rectangular tank 5.0 ft. long, 3.0 ft. deep, $2\frac{1}{2}$ ft. wide, inside dimensions?

10. Calculate the weight of a cast-iron pipe 10 ft. long, 4.0 in. bore, walls $\frac{1}{2}$ in. thick, having a circular flange at each end 9.0 ins. diameter, $\frac{9}{16}$ in. thick.

151. Pressure.—In our discussion of the mechanics of fluids we shall use the term pressure always in the sense of *intensity of pressure*, or *pressure per unit of area*. The

unit of area, unless specifically stated otherwise, is understood to be a *square inch.* Thus the expression, "Air at 30 lbs. pressure," signifies air which exerts *30 lbs. pressure on each square inch of surface,* etc.

152. Pressure in Fluids at Rest.—Suppose a small area, a, Fig. 191, to be horizontal and a depth Z below the surface of a liquid at rest. Assume all other sources of pressure removed. Then the area will be pressed vertically downward *by a force equal to the weight of the column of liquid above it.* This weight will be equal to the volume Za of liquid times its density s or Zas. If p be the *pressure* at any point of the area, then pa, or the total force on the area, must equal Zas, or $p = Zs$. Any change in the depth Z, or in the density of the liquid, will produce a corresponding change in the weight of the column of liquid resting upon a given area, and hence upon the *pressure.* Thus we have the rule:

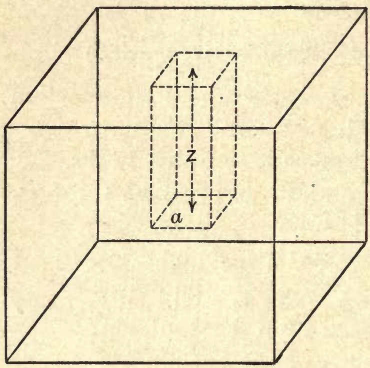

Fig. 191.

Pressure in a liquid varies with the depth and with the density of the liquid.

It is because of this greater pressure as the distance below the surface increases that dams, retaining walls of reservoirs, etc., are made thicker and stronger at the bottom, as in Fig. 200.

If the area a in the preceding illustration is not horizontal, the pressure at some parts is greater than at others, but the *pressure at any given point* in the area is as before

dependent only upon the depth of this point below the surface and upon the density of the liquid.

153. Pressure Equal in all Directions.—It is evident that the pressure at the depth Z acts equally in *every direction* up, down, and at every angle. This must be true, for otherwise the liquid, being acted on by an unbalanced force in some direction, would move, which is contrary to the supposition that the liquid is at rest, i.e., in equilibrium.

154. Direction of Pressure on the Walls of the Vessel.—It may also be seen that the pressure of a liquid on the surface of the containing vessel *is always perpendicular to that surface*, for if it were not, there would be a component of the pressure tending to urge the liquid along the surface and the liquid being free to move, would slide along the surface which, again, is contrary to the supposition that the liquid is in equilibrium.

155. Transmission of Applied Pressure through Fluids.—If we take a vessel full of water (see Fig. 192), having var-

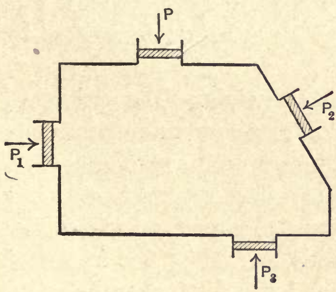

FIG. 192.

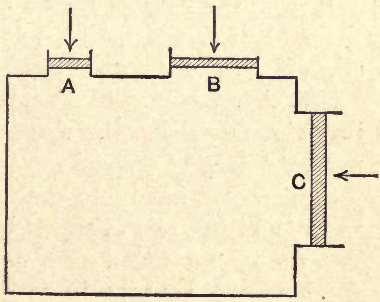

FIG. 193.

ious apertures of the same size fitted with water-tight pistons, and if one of these pistons be pressed downward with a force P an *additional pressure equal to P* will be required at each of the other pistons to preserve equi-

librium. Thus, if a force of 5 lbs. is applied to P it will be necessary to apply a force of 5 lbs. at each of the other pistons to prevent their moving, and this will be so no matter at what part of the vessel these pistons are placed. Thus we see that any increase of *pressure applied at one part is transmitted equally through the fluid.* Furthermore if, as in Fig. 193, the pistons are of different areas, that at B being twice as large area as that at A, and that at C being three times as large, then for every pound applied at A 2 *lbs. must be added* for equilibrium *at B, and* 3 *lbs. at C.* In general, the forces at the different pistons will vary directly as the areas of those pistons, or if the pistons are, as usual, of a circular cross-section, the forces will vary as the squares of the diameters. Hence we arrive at the important principle:

PASCAL'S PRINCIPLE.—*When pressure is communicated to any part of a fluid it is transmitted equally on equal areas in all directions through the fluid.*

In the illustrations above a liquid was referred to, but the student should notice that Pascal's Principle applies to all *fluids, both gases and liquids.*

156. Applications of Pascal's Principle.—The principle just given explains the action of many common and important pieces of apparatus and many useful machines.

Suppose we have two communicating vessels containing water, Fig. 194, one of which is much larger than the other. The vessels are fitted with pistons P and p, the areas of which we will suppose to be A and a. If now, weights W and w be placed on these two pistons respectively, so as to counterbalance each other, it will be found that

$$W:w=A\cdot a,$$

which is in accordance with Pascal's Principle.

We see, also, that if the piston p be pressed down through the distance S, the water contained in the smaller vessel will be caused to pass into the larger, and force up the piston P through some distance S', such that

$$a \times S = A \times S',$$

since the volume of water that is removed from one vessel is the same as that which enters the other vessel. Hence $\frac{a}{A} = \frac{S'}{S}$, that is, the simultaneous distances moved by the pistons vary inversely as their areas. Also

$$W \times S' = wS,$$

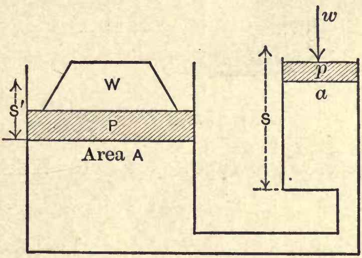

FIG. 194.

that is, the *work done by* w = *work done by* W. This is the *Hydraulic Press.*

In order to make the action of the hydraulic press continuous and to provide the motion at W necessary in practice, the smaller piston is operated as the plunger of a pump by means of which water is pumped into the reservoir. Other forms of hydraulic and pneumatic machinery, as hydraulic jacks, pneumatic drills, riveters, air-brakes, etc., act upon a similar principle of transmission of pressure.

Among common phenomena the pumping up of bicycle and automobile tires with a small hand pump, and the rise of water to the same level in a closed system of pipes, may be mentioned as instances in which Pascal's principle applies.

Fig. 195 shows the case of two open tubes of different size communicating by a flexible diaphram O, one contain-

ing mercury the other water. The pressure at O is equal in all directions (liquids in equilibrium), therefore $P_1 = P_2$.

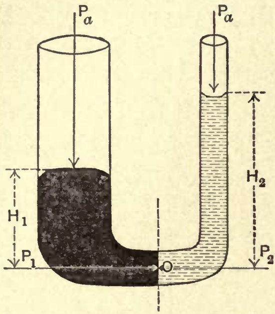

FIG. 195.

P_1 = atmospheric pressure $P_a + H_1 \times dm$. $P_2 = P_a + H_2 \times dw$, where dm and dw = density of mercury and water respectively. Therefore, $P_a + H_1 dm = P_a + H_2 dw$. Whence, $H_1 dm = H_2 dw$ and

$$\frac{H_1}{H_2} = \frac{dw}{dm},$$

or the *heights at which the two liquids stand in the tubes are inversely proportional to their densities.*

PROBLEMS

1. Two communicating vessels contain fluid, and are fitted with pistons, the diameters of which are 2 and 8 ins. respectively. If a weight of 3 lbs. is placed on the smaller piston, what weight must be placed on the larger to preserve equilibrium?

2. A narrow vertical pipe is attached to a vessel which is fitted with a piston the area of which is 200 sq.cm. If the vessel and pipe contain water, find the height of the water in the pipe when a weight of 40 kgm. is placed on the piston.

3. A cylindrical vessel contains mercury to the height of 2.0 ins. above the base and a layer of 8.0 ins. of water resting on the mercury. Find the pressure at any point in the base. Mercury is 13.6 times as heavy as water.

4. In a hydraulic press, diameter of piston in the small cylinder is $1\frac{1}{2}$ ins., diameter piston of large cylinder 16 ins. If water is forced in under pressure of 80 lbs. for the entire

area of the small piston what will be (*a*) pressure per square inch on larger? (*b*) total pressure?

5. Bicycle tire is cylindrical, $1\frac{1}{2}$ ins. inside diameter, and 5 ft. long. Opening for pump is $\frac{1}{4}$ in. diameter. If air is forced in here under a pressure of *one pound for this area*, what will be (*a*) total pressure produced on inside of tire? (*b*) Pressure per square inch? (*c*) Pressure tending *to burst tire* if atmospheric pressure is 15 lbs. to square inch?

6. What is the pressure per square inch due to water alone at the bottom of a pond 30 ft. deep?

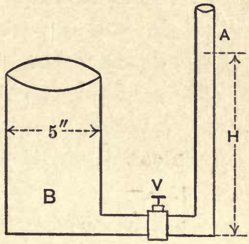

FIG. 196.

7. Pipe *A*, Fig. 196, is 1.0 in. inside diameter. It is filled with water to a height $H = 4.0$ ft. What *weight* of mercury must be placed in *B* in order that, when valve *V* is opened, the liquids will remain at the same level?

157. Whole Pressure of a Liquid on the Base and Sides of the Vessel Containing It.—The laws of fluid pressure stated in the preceding articles enable us to determine the total pressure on the walls of vessels, reservoirs, etc., whatever their shape.

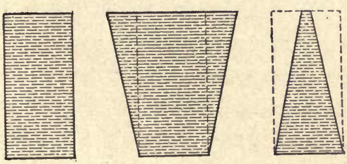

FIG. 197.

(*a*) *Whole Pressure on Base.*—Three cases may be distinguished, according as the side walls are vertical or slope from the base outward or inward. See Fig. 197.

Suppose the base in each case to be horizontal. If the sides are vertical the whole pressure is evidently on the base and the pressure is equal to the weight of the liquid.

Next, suppose the side to slope outward. In this case the pressure is *less* than the weight of the liquid. It is equal to the weight of the liquid within the dotted lines. What supports the liquid between the dotted lines and the sides?

In the third case where the sides slope inward from the base, the whole pressure on the base *is more than the weight* of the liquid in the vessel. Explain how this can be.

The general rule for the total pressure on a horizontal surface under a liquid, like the base of the vessel in either of the cases just given, is as follows:

RULE.—*The* TOTAL PRESSURE *on a horizontal surface below a liquid is equal to the area times the depth times the weight of unit volume of the liquid.*

(*b*) *Whole Pressure on a Vertical Wall.*—If the surface is vertically placed in the liquid we may find the total pressure on it as follows: Find the *average pressure* on the surface and multiply this by the total area. Pressure increases with depth, varying from zero at the top of a surface placed as *A*, in Fig. 198, to a maximum value equal to depth × density at the bottom. The average pressure for a rectangular surface is therefore the pressure at a point half way from the upper edge of the rectangle to the lower edge. The student may work out for himself the average pressure and area for the other cases illustrated in Fig. 198. Where the surfaces are not rectangles, the varying pressure is not distributed according to depth alone, and the average pressure is evidently the pressure at the *center of gravity* of the surface. It is evident from inspection that this statement applies equally to rectangles, hence *for surfaces of any form* we may apply the following general rule:

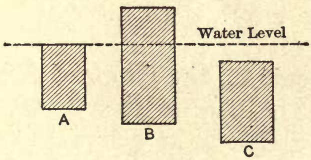

FIG. 198.

The total pressure on ANY PLANE SURFACE *below a liquid is equal to the area in the liquid times the vertical distance from the surface of the liquid to the center of gravity of the area times the weight of unit volume of the liquid.*

(*c*) *Whole Pressure on Surfaces Inclined at an Angle with the Vertical.*—Liquid pressure is *always perpendicular to the retaining surface* (Article 154), hence the general rule stated under (*b*) may be applied to these surfaces also.

158. Center of Pressure.—When a surface is immersed in a liquid, every point on the surface is subjected to a pressure at right angles to the surface.

All these pressures constitute a system of parallel forces (Fig. 199) whose resultant may be found. In our study of liquid pressure we have learned how to find the *amount* of this resultant in certain cases, but we have not considered the point on the surface at which it acts.

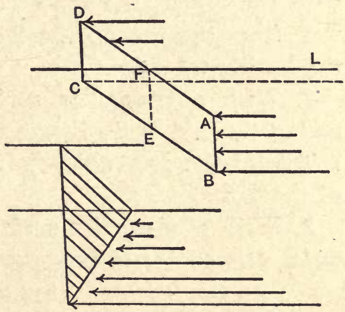

FIG. 199.

This point is called the CENTER OF PRESSURE, and is defined as the point of action of *the single force equivalent to the whole pressure* exerted by a fluid on any plane surface with which it is in contact.

If the plane surface is horizontal, the center of pressure is the same as the center of gravity of the surface. Thus, the center of pressure on a horizontal rectangle, circle, triangle, etc., is at the center of gravity of the surface and a single force (equal to the *total* pressure on the area) applied at that point will produce equilibrium.

If, however, we take the case of a rectangular surface

placed *vertically* in a liquid, since the pressures increase proportionally to the depth and the total force on the surface depends on the shape and position of the surface, the resultant can only be obtained by calculation but the *center of pressure is always below the center of gravity.* For if the pressures on equal areas on different parts of the surface were equal, the point of application of their resultant, the center of pressure, would coincide with the center of gravity. But the pressure increases with the depth and therefore the center of pressure is necessarily below the center of gravity. The exact location of this point is found to be as follows:

1. With a rectangular surface whose upper edge is level with the liquid, the center of pressure is *two-thirds* of the way from the top on the line joining the middle points of the horizontal sides.

2. With a triangular surface whose *base is horizontal, and at the level of the liquid,* the center of pressure is at the *middle* of the line which joins the vertex with the middle of the base.

3. With a triangular surface whose *vertex is level with the water,* the center of pressure is in the line joining the vertex and the middle of the base, and *three-fourths* of the way from the vertex.

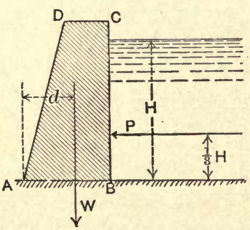

FIG. 200.

159. Stability of a Retaining Wall.—Suppose $ABCD$, Fig. 200, represents a section of a retaining wall. The resultant water pressure P will act at a distance equal to $\frac{1}{3}$ the depth of the water H from the bottom. Its tendency to overthrow the wall by tipping it over the edge at A will be measured by the moment $P \times \frac{H}{3}$. This moment will be opposed by the

moment of the weight W of the wall, or $W \times d$. Evidently, in order that the wall may stand, $W \times d$ must be greater than $P \times \frac{H}{3}$.

PROBLEMS

1. Find the whole pressure on a rectangular surface 6.0 ft. by 4.0 ft. immersed vertically in water with the shorter side parallel to and 2.0 ft. below the surface.

2. Find the whole pressure on the curved surface of a vertical cylinder which is filled with a liquid, sp. gr. 1.5, the height of the cylinder being 20 cm. and the radius of the base 7.0 cm.

3. A flood gate is 6.0 ft. wide and 10 ft. deep. What is the total pressure on the gate when the water is level with the top?

4. A box with a closed top is 30 cm. long, 20 cm. wide, and 18 cm. deep. Opening into the top of the box is a vertical rectangular tube 2×3 cm. and 40 cm. high. Both box and tube are filled with water. Find the following:

   (*a*) The pressure and the total pressure on the bottom of the box.

   (*b*) The same for the top of the box.

   (*c*) The weight of the water in box and tube together.

   (*d*) Account for the fact that the total pressure on the bottom of box is greater than the whole weight of water in the apparatus.

   (*e*) Find the total pressure on the end of the box.

   (*f*) The same for a side.

   (*g*) The same for a side of the tube.

5. What is the pressure in lbs. per sq.in. at a depth of half a mile in sea-water? Sp. gr. 1.03.

6. A cube whose edge is 20 cm. is sunk till its top, which is horizontal, is 60 cm. below the surface of water. Find the pressure on one of its vertical sides.

7. A hole 6.0 ins. square is made in a ship's bottom 20 ft. below the water line. What force is required to hold a piece of wood tightly over the hole?

8. A mill dam is 40 ft. long and the water is 15 ft. deep, and the pond is 500 ft. long. Find the force urging the dam down stream.

9. A house is supplied with water from a reservoir 240 ft. above ground. Find the pressure per square inch at a tap 25 ft. above the ground.

10. Canal lock is 60 ft. long, 20 ft. wide, 10 ft. deep.
   (*a*) Pressure per square foot on bottom?
   (*b*) Total pressure on side?
   (*c*) On gate?
   (*d*) Pressure per square foot on gate half-way down?
   (*e*) 8.0 ft. down?

11. *ABC*, Fig. 201, stands in water with *BD* in vertical line. Apex *B* is 2.0 ft. beneath surface. $AD = DC = 4.0$ ft. $BD = 10$ ft. Compute total pressure against surface.

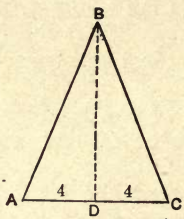

Fig. 201.

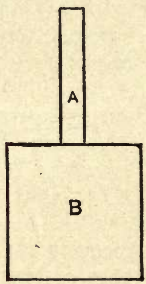

Fig. 202

12. *A* and *B* are two cylinders placed as shown in Fig. 202. *A* is 2.0 ft. high and 4.0 ins. diameter. *B* is 2.0 ft. high and 2.0 ft. diameter. When both are filled with water compute:
   (*a*) Total pressure on bottom of *B*.
   (*b*) Pressure on scale pan supporting them if cylinders weigh 25 lbs.

13. On applying a pressure gauge to a water pipe of a building it registers 75 lbs. per sq.in. How high is the supply reservoir above this point?

160. Resultant Upward Force on Bodies Immersed in a Fluid.—If we take a piece of iron weighing 1 lb. and hang it wholly immersed in water from one scale of a balance, we find that it now seems to weigh *less than 1 lb.* The difference in the weight in the two cases is due to the resultant upward pressure of the water on the iron.

In Fig. 203 we have a rectangular block immersed in water with one surface horizontal. It is evident that the pressure on one vertical side of the block is just equal to that on the other side, and that there will be in general no resultant horizontal pressure on the block. The resultant upward pressure is easily found as follows: The pressure on the top of the block is evidently that of a column of the liquid having AB for a base and AE for height, while the pressure on the bottom of the block is that on a column of the liquid having DC for a base and DE for height. The resultant upward pressure must be equal to the difference between the weight of these two columns, i.e., to the weight of a column of liquid $ABCD$, that is, *to the weight of the liquid displaced.* If the body be of irregular shape, a more general proof must be employed, but the result is the same. Hence,

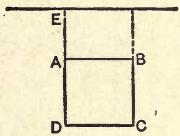

Fig. 203.

Archimedes' Principle.—*A body immersed in a liquid loses a weight equal to the weight of the fluid displaced.*

It is evident that Archimedes' principle furnishes a convenient method of determining the *Specific Gravity* of a body and also *the volume of irregular bodies.* Thus suppose a body weighing 80 gms. in air, weighs only 60 gms. when

immersed in water. The loss of weight is 20 gms. and according to Archimedes' Principle, this is *the weight of a volume of water equal to the volume of the body.*

$$\text{Its specific gravity} = \frac{\text{weight of body}}{\text{weight of equal vol. water}} = \frac{80}{20} = 4.$$

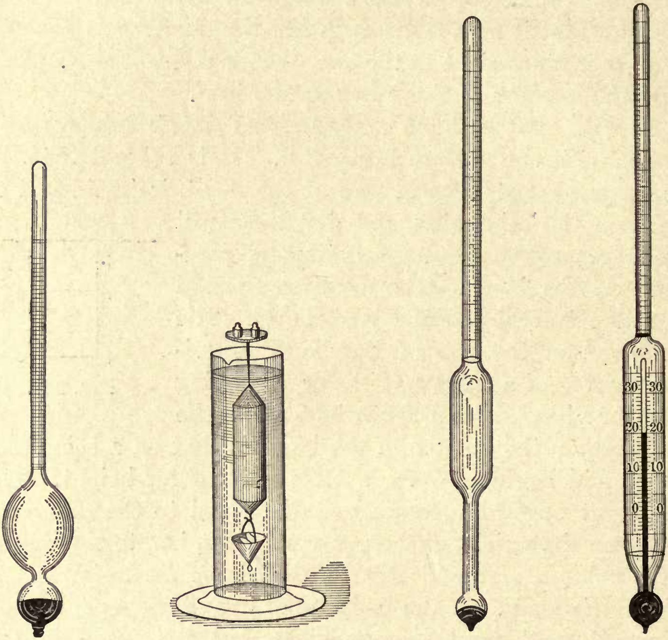

FIG. 204.

And since 1 cc. of water weighs 1 gm., if a volume of water equal to that of the body weighs 20 gms. the volume of the body is 20 cc.

Archimedes' Principle applies to bodies immersed in a gas as well as to those in a liquid. Hence the *real weight*

of all bodies (that is, their weight in a vacuum) is greater than their weight in air, which may be called their apparent weight, by the weight of the air displaced.

Do two bodies having the same apparent weight have always the same real weight? When do they? Give the rule for finding the real weight of a body from its apparent weight and the other necessary data. Explain the lifting power of a balloon.

161. Hydrometers.—Floating bodies sink in the liquid until they displace their own weight. The resultant upward force then equals the downward force of gravity and the bodies are in equilibrium. It is evident that a floating body must sink deeper in a light liquid than in a denser in order to displace its own weight; or, in other words, the volume of two liquids displaced must be inversely as the densities. This principle is the basis of the usual forms of HYDROMETERS, or instruments which indicate the density of liquids by the scale readings on their stems. Common types of hydrometers are shown in Fig. 204. These instruments are calibrated by placing them in liquids of known density and marking upon the stem the point to which they sink, that for pure water being marked 1.000. To provide greater sensitiveness, a single instrument is designed to measure a small range of densities only. Special instruments are thus made for testing alcohol, milk, acids, etc.

PROBLEMS

1. A piece of glass weighs 24 gms. in air and 16 gms. in water. Find its specific gravity.

2. A block of wood is placed in a vessel just full of water. It floats half submerged and 100 cc. of water run out. Find the weight, volume, and specific gravity of the wood.

3. A barge with vertical sides, floating in fresh water, is 30 ft. long and 20 ft. wide. An elephant is driven upon the barge and the barge is then found to have sunk 2 ins. Find the weight of the elephant.

4. A man weighs 75 kgm. and his volume, exclusive of his head, is 72,000 cc. How many cubic centimeters of cork (sp. gr. 0.25) are required to keep the man floating with his head above water?

5. A body weighs 300 gms. in air, 248 gms. in water, and 206 gms. in an acid. What is the sp. gr. of the acid?

6. A piece of wood weighs 120 gms. in air. A piece of lead weighs 30 gms. in water. Both together weigh 20 gms. in water. Find the sp. gr. of the wood.

7. What fractional part of an iceberg (sp. gr. 0.918) will float above sea-water (sp. gr. 1.03)?

8. What will be the loss of weight in water of 2 cu.ft. of stone? Of 2 cu.ft. of iron? Of 2 cu.ft. of wood?

9. Find the volume of a body that weighs 350 gms. in a vacuum and 225 gms. in water.

10. A piece of metal weighs 36.0 lbs. in air and 32.0 lbs. in fresh water. What will it weigh in sea-water of density 1.03?

11. A cylindrical post 30 ft. long and 8.0 ins. in diameter, has a weight of 880 lbs. It has 6.0 ft. of its length projecting vertically out of water. What force is required and in what direction, to keep it in this position?

12. A glass tube 100 cms. long holds 210 gms. of mercury. Compute internal diameter of the tube.

13. An iron ball weighing 105 gms. is suspended under water. Tension in suspending string?

14. An iron weight of 1.0 kilo is to be a cylinder 5.0 cms. thick. What diameter must it be made?

GASES

162. Gases Compared with Liquids.—Gases, like liquids, have weight and such perfect freedom from molecular friction that they obey the laws of Pascal and Archimedes.

Gases, however, differ from liquids in three important respects:

(1) They behave as if their molecules repelled each other.

(2) They are very compressible.

(3) They expand much more rapidly when heated.

The proof of the first of these statements is seen in all the familiar cases where a gas expands, as its pressure is reduced. A gas must be kept in a vessel enclosed on all sides or it will expand indefinitely.

The proof of (2) is also given by many common experiments, some of which will presently be considered.

163. The Density of a Gas Depends upon Both its Pressure and its Temperature.—If no specifications are given, it may usually be assumed that *Normal Conditions* are intended, i.e., a pressure of 30 ins. or 76 cm. of mercury and a temperature of 0° Centigrade, 32° Fahrenheit. The specific gravity of a gas is usually referred to that of dry air as a standard.

164. Atmospheric Pressure.—That the air has weight is a fact not known till about 1620, when the work of Galileo, Torricelli, Pascal, Guericke, and others established the fact. The weight of the air may be demonstrated in the same way as the weight of anything else, that is, by weighing a vessel full of air and then the same vessel empty, and observing the difference. The weight of the air has the most important consequences upon the conditions not only of human life, but also upon the conduct of very many practical operations and scientific experiments. All the phe-

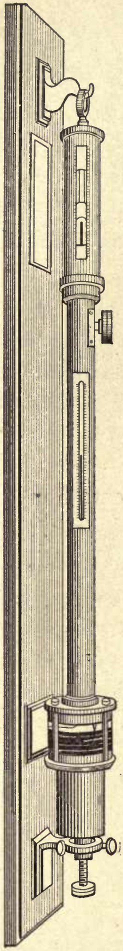

FIG. 205.—Standard barometer.

nomena commonly spoken of under the term "suction," are due to inequalities of atmospheric pressure, i.e., the rise of water from the cistern into the barrel of a common pump. The operation of siphons, of various types of air-pumps, etc., depends upon atmospheric pressure.

165. The Barometer.—Torricelli (an Italian, died 1647) devised an experiment to show the weight of the air, which may be performed as follows: Take a glass tube at least 34 ins. in length, open at one end and closed at the other. Fill it carefully with mercury and, placing the thumb over the open end, invert the tube with this end under the surface of some mercury contained in a cup. On removing the thumb, the mercury will be found to sink somewhat and will soon come to rest with its surface at a level about 30 ins. higher than the level of the mercury in the cup. Now it is evident that the pressure of the column of mercury in the tube is equal to the height of the column times the weight of a unit volume of the mercury, or about 15 lbs. per sq.in. The atmospheric pressure on the surface of the mercury in the cup therefore must also be about 15 lbs. per sq.in. to maintain the mercury in the tube. And since the pressure at the mouth of the tube must always be the same in all

directions, if atmospheric pressure increases, the mercury must rise in the tube until its weight becomes sufficient for equilibrium; and if atmospheric pressure decreases, the column of mercury must sink for a similar reason. Torricelli's apparatus is therefore a simple BAROMETER, or instrument for measuring atmospheric pressure.

The common form of standard barometer is shown in Fig. 205. As the scale for reading the height of the column of mercury is rigidly attached to the case, the first operation in determining atmospheric pressure with this form, is to set the surface of the mercury in the cistern at *zero* of the scale. This is done by means of a screw, which raises or lowers the flexible bottom of the cistern until the mercury is just in contact with the tip of the ivory point which marks the zero of the scale.

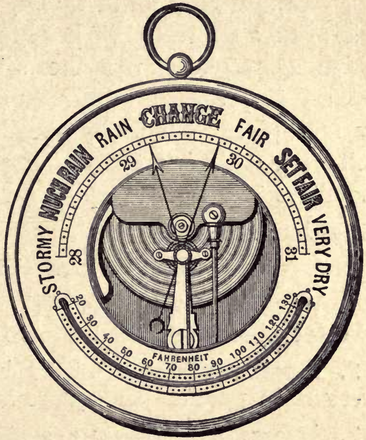

FIG. 206.—Aneroid barometer.

The so-called *Aneroid Barometer*, Fig. 206, consists of a thin metallic box tightly closed on all sides, and having a top of corrugated form which is easily compressed to a slight degree. The air in the box is slightly exhausted so that the variations of the pressure of the air outside cause the top of the box to move in or out through small distances. A suitable set of small gears and levers transmits these motions to a pointer moving over a scale. The scale is marked off in the first place by comparing the aneroid with a standard mercury barometer, and the instrument requires to be frequently so compared to be reliable.

166. Standard Pressure.—As has been stated, the mercury will stand in a barometer tube at a height of about 30 ins. or 76 cms. This is, however, subject to considerable variation due to the changes in weather, etc. "Standard Pressure" means a pressure of 30 ins., or what is almost exactly the same, 76 cms. of mercury at a temperature of 0° C. This is the pressure to be assumed in the absence of any specifications, and is the pressure to which all scientific data are reduced in tables, etc.

Pressures expressed in heights of the column of mercury which atmospheric pressure will support may readily be reduced to pressures in lbs. per sq. in., or grams per sq. cm. by multiplying height by the density expressed in proper units. (See Article 152.) Thus, 30 ins. of mercury corresponds to 14.7 lbs. per sq. in. (in round numbers, 15 lbs. per sq. in.).

PROBLEMS

1. What fractional part of the air has been removed from a vessel when the pressure has fallen from 74 to 25 cms.?

2. Barometer stands at 29 ins. Compute atmospheric pressure in pounds per square inch.

3. Atmospheric pressure sustains 30.2 ins. of mercury. How high a column of water will it sustain?

4. A pair of Magdeburg hemispheres are 6.0 ins. in diameter. Pressure inside is .50 in. of mercury, outside 30 ins. Compute force required to pull them apart.

5. If in ascending a mountain the barometer falls from 76 to 51 cms., find the decrease in total pressure on an area of 1 sq.ft.

6. If a man's body has a surface of 18 sq.ft., find the total pressure upon him in tons.

7. Find the specific gravity of air when the barometer is at 58 cms., if it is .0013 when the barometer is at 76 cms.

8. Find the greatest height that water can be carried

over a siphon at the top of a mountain where the barometer stands at 58 cms.

9. A hollow sphere has an external diameter of 6.0 ins. What is the total force due to atmospheric pressure holding any two hemispheres together? Answer in pounds.

10. The piston of a steam engine has a diameter of 10 ins., and the steam exerts a pressure upon it of 5 atmospheres. Find the effective force when the other side of the piston is exposed to the atmosphere.

11. If the density of air, like that of water, were uniform and equal to that of the air at the sea level, how high would the atmosphere extend? Assume the height of a water barometer to be 34 ft.

167. Boyle's Law.—We have seen that the characteristic qualities of a gas are its expansibility and compressibility.

Experiment shows that if the volume of a gas be *doubled*, its *pressure is reduced one-half;* if the volume be made *one-third* as great, the *pressure* will become *three times* as great, etc. In general, we have the statement known as

BOYLE'S LAW.—*The volume of a gas varies inversely as its pressure, when the temperature remains constant.* (This is sometimes called Marriotte's Law.)

As the *mass* of the gas remains the same, it follows that the density of a gas must increase as the volume decreases, and vice versa. Hence Boyle's Law may also be stated thus:

The pressure of a gas is proportional to its density, the temperature remaining constant. Or,

The weight of gas in a given volume at a constant temperature is directly proportional to its pressure.

To state this law in symbols, let V_1 and V_2 be the volumes at pressures P_1 and P_2 respectively; also let d_1 and d_2 be the corresponding densities. Then

$$\frac{V_1}{V_2}=\frac{P_2}{P_1} \quad \text{or} \quad P_1V_1=P_2V_2, \quad \text{also} \quad \frac{d_1}{d_2}=\frac{P_1}{P_2}.$$

For a given constant volume with varying pressures, $\frac{W_1}{W_2}=\frac{P_1}{P_2}$.

It should be added that the laws stated above are *not absolutely obeyed by any gas.* The variations, are however, so small as to be discernable only by the most delicate experiments and in all practical work and most scientific investigations it is considered that all gases follow Boyle's Law precisely. A gas which obeys Boyle's Law and another law known as Charles' Law (to be stated under the subject of Heat), is known as a PERFECT GAS. The idea is of value in theoretical discussions. As just stated, in practical work all gases are assumed to be Perfect Gases, though none of them is so in fact.

168. Dalton's Law.—This is an extension of Boyle's Law for a mixture of different gases. If several gases which do not act chemically on each other are placed in a vessel, the pressure on the sides of the vessel is the sum of the pressures due to the different gases. Thus if the pressures of the different gases are P_1, P_2, P_3, P_4, etc., the intensity of the total pressure exerted by the mixture is $P_1+P_2+P_3+P_4$, etc.

Dalton's Law may be stated as follows: *When a mixture of several gases at the same temperature is contained in a vessel, each gas produces the same pressure as if the others were not present.*

PROBLEMS

1. A quantity of air at 0° C. and under a pressure of 95 cms. measures 100 cc. Find its volume at 76 cms. pressure.

2. What will 1 liter of air weigh at 0° C. under a pressure of 4 atmospheres, if 1 liter of air at 0° C. and 76 cms. pressure weigh 1.29 gms.?

3. A gas inclosed in a cylinder with a freely moving piston has a volume of 1.23 cu.ft. when the barometer is at 29.5 ins. To what height must the air pressure fall in order that the volume of the gas may become 1.34 cu.ft. at the same temperature?

4. What fractional part of the air in the receiver of an air-pump has been removed when the pressure within has fallen from 30 to 5.6 ins.?

5. A diving bell is lowered till its lower edge is at depth of 15 ft. below the surface of the water. The barometer pressure at the time is 30 ins. The space within the cylindrical bell is 12 ft. high. At what point within the bell will the water level stand?

6. If a room contains 23 lbs. of air at a pressure of 75 cms., how many pounds will it contain at a pressure of 72 cms.?

7. A cylinder 40 cms. long is fitted with a piston 2.0 cms. thick. The piston is placed at the center of the cylinder and both sides of it are opened to the air and then closed. The barometer stands at 74 cms. The piston is then forced to a position such that its center is 9.0 cms. from one end of the cylinder. What is then the pressure in each side?

8. The mouth of vertical cylinder 18 ins. high filled with air is closed by piston weighing 6.0 lbs. and area 6.0 sq.ins. If the piston is allowed to fall, how far will it descend?

9. 1000 cc. of gas at 760 mm. pressure will have what volume under 500 mm.? Under 1800 mm.?

10. 100 cu.ft. of air at pressure of 15 lbs. per sq.in. is compressed to 36 cu.ft. at the same temperature. Pressure then?

11. A cylinder 40 cms. long inside is fitted with a tight piston. When the gas inside is at atmospheric pressure, 75 cms., the inner face of the piston is 35 cms. from the closed end. The piston is pushed in so that the space within is successively 30, 25, 20, 15, 10, and 5 cms. long. Calculate the pressures at these positions and plot the

results on a pressure-volume diagram. (Use volumes as abscissæ.)

12. Plot a pressure volume-diagram from the following data, using pressures as ordinates and volumes as abscissæ.

Pressures. Lbs. per Sq. In.	Volume. Cu. In.	Pressures. Lbs. per Sq. In.	Volumes. Cu. In.
200	100	100	200
180	111	80	250
160	125	60	234
140	143	40	500
120	167	20	1000

169. Measurement of Gas Pressures.—Manometers or pressure gauges are instruments for measuring the pressure of gases or vapors in closed systems. The ordinary commercial form of vacuum and pressure gauges is shown in Fig. 207.

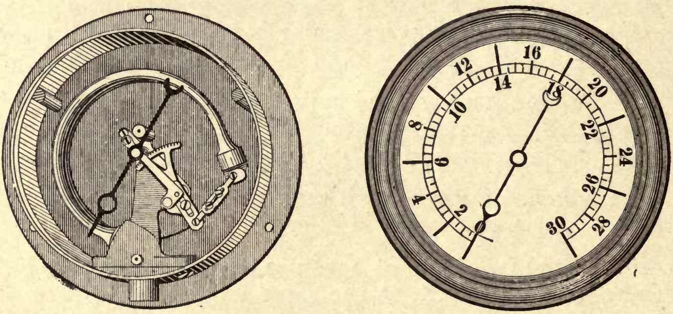

FIG. 207.—Common type of pressure gauge.

Two forms of simple manometers, the open and the closed tube types, are shown in Fig. 208.

In use, type one is filled with some liquid of known density, and the end *a* is connected with the closed ves-

sel or pipe, the pressure within which is to be determined. The difference in level of the liquid in the two arms indicates the difference between the pressure exerted by the atmosphere and the pressure exerted by the confined fluid within the vessel. It may therefore be used for pressures either greater or less than atmospheric pressure. The actual pressure is obtained by computation from the barometer reading and the observed difference in reading of the manometer columns.

It should be carefully noted that the difference between the levels in the manometer gives, when reduced to pressure units, the EFFECTIVE PRESSURE within the vessel, and to this must be added the atmospheric pressure to obtain the TOTAL PRESSURE. It is the former of these that is given by the ordinary steam gauge as used on boilers, etc.

Type II is used for the measurement of higher pressures. The end *b* is closed and contains a column of air above the *mercury* in the tube. In using such a manometer, readings are first taken of the volume of air (usually expressed simply as lengths of tube occupied by the air), and the difference of levels of the mercury when the end *a* is open to the air. Connection is then made at *a* with closed vessel. The pressure of the fluid in this vessel will change the level of the mercury. Readings are then taken of the volume of the air and the difference of levels of the mercury in the tubes. From the two sets of readings the volume of the air and the pressure upon it (the reading of the barometer being known) in the beginning, and the

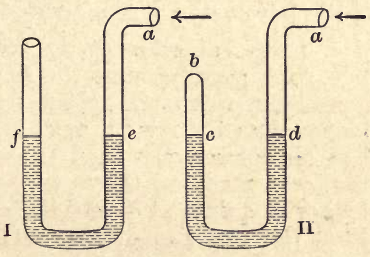

FIG. 208.

volume of the air under the unknown pressure of the vessel are known. The unknown pressure may then be computed by the application of Boyle's Law. This, corrected for the difference of level of the mercury, will give the pressure of the closed vessel.

PROBLEMS

1. The pressure of gas in a main is found to cause the water in one arm of an open-arm manometer to stand 2.4 ins. higher than in the other. Find the pressure of the gas in pounds per square inch.

2. An open-arm manometer is filled with mercury and when it is connected to a certain cylinder of gas the mercury in the open arm rises to a position 4 cms. above the level of the mercury in the closed arm. Find the pressure indicated in kilograms per square centimeter.

3. In the preceding problem, what is the total pressure if the barometer at the time stands at 73 cms.?

4. In the diagram of the closed-arm manometer on the preceding page, assume the following readings: Before the connection was made to the vessel the level *d* was 4.0 cms. higher than the level *c*, and the barometer stood at 75 cms. The air column was then 10 cms. long. After the connection was made to the vessel, the mercury stood 2.3 cms. higher at *c* than at *d*, and the air column was 6.1 cms. long. Find the pressure of the fluid in the vessel (*a*) in centimeters of mercury; (*b*) in atmospheres; (*c*) in feet of water; (*d*) in pounds per square inch.

5. Data similar to the preceding problem to find the same items. Before connection, *d* is 5.2 cms. higher than *c*; air column 15 cms.

6. I wish to measure the water pressure at a tap and connect with an open mercury manometer. When the tap is opened, mercury is forced to height of 110 cms. in the open arm. Express "head" in feet.

APPENDIX

USEFUL NUMBERS

$$\pi = 3.1416 = \frac{\text{circumference}}{\text{diameter}}$$

$$\pi^2 = 9.8696; \quad \frac{1}{\pi} = .3183.$$

$$\text{Area of circle} = \pi r^2 = \frac{\pi d^2}{4} = .7854 d^2.$$

$$\text{Surface of cylinder} = 2\pi r l + 2\pi r^2.$$

$$\text{Volume of cylinder} = \pi r^2 l.$$

$$\text{Surface of sphere} = 4\pi r^2.$$

$$\text{Volume of sphere} = \frac{\pi d^3}{6} = \frac{4\pi r^3}{3}.$$

METRIC-ENGLISH EQUIVALENTS

1 cm.	= .39 in.	1 in.	= 2.54 cms.
1 m.	= 39.37 ins.	1 ft.	= 30.48 cms.
1 m.	= 3.23 ft.	1 ft.	= .305 m.
1 km.	= .6 mile	1 mile	= 1.60 km.
1 gm.	= .035 oz. (avoir.)	1 oz.	= 28.35 gms.
1 kgm.	= 2.204 lbs. (avoir.)	1 lb.	= 453.6 gms.
1 sq. cm.	= .154 sq. in.	1 sq. in.	= 6.45 sq. cms.
1 cu. cm.	= .061 cu. in.	1 cu. in.	= 16.39 cu. cms.

WEIGHTS (Approximate)

1 cu. ft. of water weighs..................	62.5 lbs.
1 cu. ft. of water weighs..................	1000. ozs.
1 cu. in. of water weighs..................	.036 lb.
1 cu. ft. of air ("Standard") weighs......	.0817 lb.
1 cu. cm. of water weighs	1.0 gm.
1 cu. cm. of air ("Standard") weighs	.00129 gm.
1 gallon (231 cu. ins.) water weighs.......	8.32 lbs.
1 cu. in. cast iron weighs.................	.26 lb.
1 cu. ft. cast iron weighs.................	450. lbs.
1 cu. in. brass weighs....................	.30 lb.
1 cu. in. copper weighs...................	.32 lb.
1 cu. in. lead weighs.....................	.41 lb.
1 cu. ft. sandstone weighs................	144. lbs.
1 cu. ft. slate weighs....................	175. lbs.
1 cu. in. mercury weighs..................	.49 lb.

UNITS OF FORCE, WORK, POWER, ETC.

Value of "*g*" at New York.............	980.2 cm/sec^2
Value of "*g*" at New York.............	32.16 ft/sec^2

(N.B.—980 cms. or 32 ft. are close enough for ordinary use.)

1 dyne	= .00102 gm.
1 poundal	= 1.3825 dynes.
1 ft.-lb.	= 1.356×10^7 ergs.
1 joule	= 10^7 ergs.
1 horse-power	= 33,000 ft-lbs/min.
1 horse-power	= 550 ft-lbs/sec.
1 horse-power	= 7.46×10^9 ergs/sec
1 horse-power	= 746 watts.
1 watt	= .00134 horse-power.
1 watt	= 10^7 ergs/sec. = 1 joule/sec.

MECHANICAL EQUIVALENTS OF HEAT

1 gm. of water heated 1° C. $=4.2\times10^7$ ergs.
1 lb. of water heated 1° C. $=1400$ ft.-lbs.
1 lb. of water heated 1° F. $=778$ ft.-lbs.

The combustion of 1 lb. of coal produces about 14,000 B.T.U.

COEFFICIENT OF ELASTICITY (TENSION AND COMPRESSION)

Steel	30,000,000 lbs/in^2.
Wrought iron	30,000,000 lbs/in^2.
Copper	15,000,000 lbs/in^2.

(*Note.*—For torsion use $\frac{2}{5}$ of above.)

TENSILE STRENGTHS

(N.B.—These values are to be regarded as only averages; there are wide variations according to hardness, composition, etc.)

Steel	100,000 lbs/in^2.
Wrought iron	55,000 lbs/in^2.
Copper (wire)	55,000 lbs/in^2.
Brass (wire)	50,000 lbs/in^2.

FACTORS OF SAFETY (MERRIMAN)

Material.	For Steady Load (Buildings).	For Varying Load (Bridges).	For Shocks (Machines).
Timber	8	10	15
Brick and Stone	15	25	35
Cast Iron	6	15	20
Wrought Iron	4	6	10
Steel	5	7	15

SIGNIFICANT FIGURES

The results of all experimental work should be so expressed as to indicate as nearly as possible the degree of precision with which the work was performed. It is evident that all numbers that are obtained as the result of measurements are limited in precision by the nature of the apparatus employed, by the care used by the observer, the size of the units, etc. In this respect, then, such quantities are quite different from the pure numbers of absolute value as employed in ordinary arithmetical operations.

The student must observe carefully the following rules in all laboratory work and in reports.

I. Recording Readings

In general, scales, etc., are to be read to tenths of the smallest divisions marked on the instrument. The last figure entered in the record is thus assumed always to be an estimation and *therefore doubtful.*

Example 1.—15.57 cms. means that a distance was measured by a scale subdivided to *millimeters,* and that the observer *estimated* the seven; thus the distance is known to be between 15.5 and 15.6, and *estimated* to be $\frac{7}{10}$ths of the way between these two values. It is misleading, and furnishes only a clue to what we *actually know* about this distance to record it as 15.6 or 15.570 cm.

Example 2.—A distance is being measured with a rule subdivided to *tenths of inches.* The observer finds the distance to be as nearly exactly seven inches *as he can distinguish.* This should be recorded *7.00 in.* (not 7.0 or 7 ins.). Why?

Example 3.—A balance is capable of weighing an object to .01 gm. and .001 gm. can be estimated. Notice the correct records for following:

Eight gms.	8.000 gms.
Eight and $\frac{1}{2}$ gms.	8.500 gms.
Eight and $\frac{7}{100}$ gms.	8.070 gms.
Eight and $\frac{8}{1000}$ gms.	8.008 gms.
Eight-tenths gm.	.800 gms.

In general a series of readings made with the same instrument should all show the same number of places filled in to the right of the decimal point *even if one or all these* places are *zeros.* Why?

It is often convenient to express *in decimal form* readings taken from scales divided into halves of units, quarters, eighths, etc. In all such cases, retain only as many places in the decimal as correspond approximately to the same degree of precision as would be expressed by the fraction, i.e., to the nearest half unit, to the nearest quarter, etc. If the first decimal figure rejected is 5 or greater, call the preceding figure one larger than before.

A study of the following table should make this clear:

$\frac{1}{2}=.5$	$\frac{1}{8}=.1$	$\frac{3}{8}=.4$	$\frac{2}{3}=.7$
$\frac{1}{3}=.3$	$\frac{1}{16}=.06$	$\frac{5}{8}=.6$	$\frac{3}{4}=.8$
$\frac{1}{4}=.3$	$\frac{1}{32}=.03$	$\frac{7}{8}=.9$	$\frac{3}{16}=.19$
$\frac{1}{5}=.2$	$\frac{1}{64}=.02$	Etc.	Etc.
$\frac{1}{6}=.2$			

II. Use of Data in Calculations

Wherever the figure following the doubtful (last retained) figure is 5 or greater than 5, increase the doubtful figure by unity. Thus, if but three figures are to be kept, 15.75, 15.76, 15.77, 15.78, and 15.79 would all be entered 15.8.

Notice especially that the *location of the decimal point has nothing to do with significant figures.* Thus, 275, 27.5,

2.75, .275, .0275, .00275, etc., are all results expressed to some degree of precision, and in each there are *three and only three* significant figures, the 5 being the doubtful figure in each.

Averages.—In averaging a series of determinations, in general, retain in the result the same number of significant figures as in any one item.

But if a *large* number of items closely agreeing with each other are averaged, the result may contain *one more* significant figure than any item.

Multiplication.—After the operation, keep in the result as many figures, *counting from the left,* as there are significant figures in the factor having the *lesser number* of significant figures.

Division.—In dividing one number by another, keep in the quotient as many figures as there are significant figures in the number having the lesser number of significant figures. Continue the divisions only far enough to determine the required figures.

Note on Multiplication and Division.—Ciphers immediately following the decimal point, *when there are no figures to the left of the point, do not count* as significant. Study the following examples:

(*a*) $15.75 \times 3.08 = 48.5.$
(*b*) $.096 \times .096 = .0092.$
(*c*) $.1523 \times .00113 = .000172.$
(*d*) $720 \times 3.1 = 2200.$
(*e*) $900 \times 800 = 720000.$

In (*e*) only the *first cipher* is significant. It is necessary to add the other three to express the number properly.

(*f*) $325.6 \div 72.5 = 4.49.$
(*g*) $.0007859 \div 157 = .00000506.$

Use of Pure Numbers, Constants, etc.—In using pure numbers and constants such as (3.1416), .7854, etc., do not employ more figures than there are significant figures in the values depending on experiment which are used with them in the same calculation. Thus if the diameter of a circle is measured as 4.51, the area is $4.51 \times 4.51 \times .785 = 15.9$. The use of more numbers in the constant lengthens the computation and gives no better result. Why?

CURVES

Coördinate Axes.—The position of a point in space may be fixed by reference to two known straight lines intersecting at right angles in the same plane as the point (*OX* and *OY* of the Fig. 209). Such lines are known as *coördinate axes.*

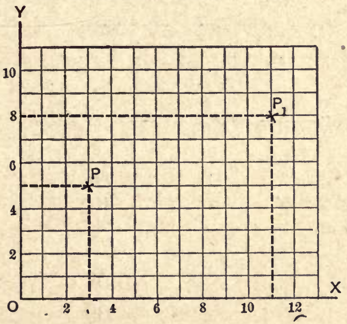

Fig. 209.

The horizontal line (*OX*) is known as the "*axis of abscissæ*" or "*X axis,*" the vertical line (*OY*) as the "*axis of ordinates*" or "*Y axis.*" The point of intersection *O* is called the *origin*. The *abscissa* of a point is its horizontal distance from *OY*; its *ordinate* is its vertical distance from *OX*. These given, the position of the point is determined. Thus *P* is that point which has an abscissa of 3, an ordinate of 5, P_1 the point which has abscissa of 11, ordinate of 8, etc.

For convenience, squared or "cross-section" paper is used for work of this kind

Curves.—A succession of related points may be connected by a smooth line, thus constituting a "*curve.*" Such curves are frequently the most convenient and the clearest way of representing a physical law, corrections for errors of apparatus, etc. Suppose, for example, that it is desired to show the relation between the stretch of a wire and the stretching loads producing it, data being as follows:

Load.	Increase in Length.
5 lbs.	.010 inch
10 "	.019 "
15 "	.030 "
20 "	.040 "
25 "	.051 "

Taking the stretching loads, expressed in some convenient scale of lengths, as ordinates, and the corresponding elongations similarly expressed, as abscissæ, a series of points may be located as just explained, and through these a smooth line may be drawn. Inspection of the curve thus produced (Fig. 210) will show at a glance what could be obtained from the figures only on more extended analysis. The law, "Elongation is proportional to the load applied," is seen immediately, from the nature of the curve.

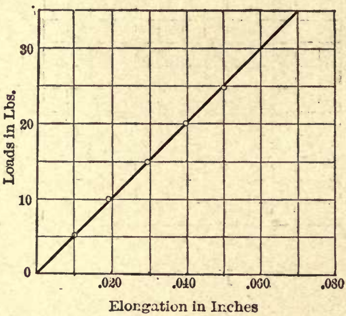

FIG. 210.

Had the curve turned continuously more and more toward either the X or the Y axis, showing in one case a progressive increase, in the other a progressive decrease in

elongation with increase of load, or had, at any time, a sudden change from the conditions which had previously existed occurred, these factors would have been brought to the attention as quickly.

When also, as here, the great majority of points lie along a straight line (or, as in some cases, along a smooth curve), any experimental errors of measurement (as in the elongations for loads of 10 and 25 lbs.), will be shown at once by the fact that these points lie slightly off the line. In all such cases, the curve should be drawn as nearly as may be through all points, and leaving as many points on one side as on the other.

The student must in all cases use his judgment in drawing the curve and consider the conditions of the experiment and the general physical law illustrated.

It is not necessary, and indeed often not advisable, that ordinates and abscissæ be expressed in the same scale. Of course, for the same curve all abscissæ must be in one scale, and all ordinates in one scale. In general, the scale adopted should be that most convenient for the particular values which will at the same time give a curve as large as the paper will permit.

One, two, five, or ten units to a square will be found the best. Avoid the use of three or seven units per square, or other inconvenient subdivisions.

GENERAL DIRECTIONS FOR CURVE SHEETS

(1) The curve must be done neatly in India ink.

(2) Heavy lines one inch in from the margin on the ruled portion are to be taken as axes, except where all the paper is necessary for the curve. The origin, i.e., the intersection of vertical and horizontal axes, should be at the lower left-hand corner. The paper may be used with

either longer or shorter side as vertical axis, according to needs of the curve.

(3) The scale on which the curve is plotted should be so selected as to make the curve as large as possible.

(4) Each axis should be marked with the quantity which it represents, and with the unit in which these quantities are expressed, e.g., "loads in pounds per square inch," "elongations in inches," etc. These titles should be lettered upon the *ruled paper between margin and axes.*

(5) Each *half-inch line* along both vertical and horizontal axes should be marked with the value which it represents. *No other figures* are to be used in locating the curve.

(6) The points fixing the curve are to be located by a small dot around which is drawn a small circle with a pair of dividers.

(7) The curve should usually be a *smooth line* drawn as nearly as possible through all points. It will represent the most probable value of the observations, and any single point lying at a distance on either side of the line will usually be a result of error in observations. Of course judgment must be used in drawing this conclusion, and the conditions of the experiment and the nature of the related quantities of the curve must always be taken into account.

(8) The name of the student and the date should be placed at the bottom of the sheet, at the right, in *small* letters.

(9) The title of the curve should be stated in the lower right-hand portion of the curve sheet unless this interferes with the curve; in which case the lower left-hand or the upper right-hand portion should be used.

(10) If more than one curve is drawn on the same paper for comparison, etc., use the *same origin and scales* for all.

Distinguish the curves by the title printed along the curve, or by lines of different colors.

(11) All titles, explanations, etc., must be in lettering, and no handwriting should appear upon the curve sheet.

THE EQUATION OF A STRAIGHT LINE

It is often desired to find the equation that corresponds to a given line (straight or curved) plotted on squared paper. In this course it will not be necessary to obtain the equation of a curved line. A simple method for the equation of a straight line follows:

Let AB, Fig. 211, be a line plotted as usual on the axes OX and OY, and meeting the axis of Y at the point A. (If the line as first drawn does not cut the axis of Y it must be extended till it does so.)

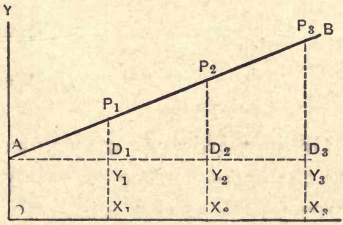

FIG. 211.

At the point A draw a line parallel to the axis of X. Choose any point on the line as P_2, and draw its ordinate y_2. x_2 is the abscissa of this point. We desire to obtain an equation that will give us the relation between the abscissa and the ordinate for this and every other point on this line.

We notice first that the ordinate y_2 equals the intercept OA on the Y axis, plus P_2D_2, or

$$y_2 = OA + P_2D_2.$$

Also, $$y_1 = OA + P_1D_1,$$

$$y_3 = OA + P_3D_3,$$

and so on for every point on the line.

The value of the *intercept* OA may now be read from the curve. Suppose in the given case $OA=8$. Next read from the curve *values* of the *altitude* and *base* of *any triangle* whose hypothenuse is some part of the line AB. These values are to be expressed in units of the respective scales used in plotting X and Y and *not* as actual lengths in inches. The triangle AP_2D_2 will serve. Suppose $P_2D_2=4$ and $AD_2=10$ in the given case. Then $\frac{P_2D_2}{AD_2}=.4$. But $AD_2=x_2$, therefore $P_2D_2=.4x_2$.

If we had used other triangles we should have obtained the *same ratio* between altitude and base, and thus,

$$P_1D_1=.4x_1,$$

$$P_2D_2=.4x_2,$$

$$P_3D_3=.4x_3.$$

Or, in words, we may now say that any ordinate equals the intercept on the Y axis plus .4 of the abscissa for the same point. Let x and y be the coördinates of *any point on the line* AB; then

$$y=8+.4x,$$

which is the equation desired.

The ratio $\frac{P_2D_2}{AD_2}$ is sometimes called the *slope* of the line.

We may now state the general *rule* as follows:

Rule.—The equation of a straight line is formed by putting y equal to the intercept on the axis of Y plus the slope times x. If intercept $=a$, and slope (ratio) $=m$, we have,

$$y=a+mx.$$

NOTE.—The student will notice that the equation just given is perfectly general. If the line cuts the axis of Y *below* the origin, the intercept will be a *negative* term and the equation will be of the form $y=-a+mx$. If the line slopes so that an increase in the value of the abscissa causes a decrease in the value of the ordinate, then m will be a negative quantity, $y=a-mx$. It is possible, of course, that both a and m may be negative at the same time, as $y=-a-mx$. The student should draw and consider carefully lines to illustrate each case.

EXERCISES

1. Locate following points:

Point.	Abscissa.	Ordinate.
(*a*)	5	3
(*b*)	7	10
(*c*)	5	8
(*d*)	0	12
(*e*)	5	5
(*f*)	9	0

2. Measure carefully the lengths of 9 ins., 7 ins., 5 ins., 3 ins., respectively in metric units. Make each measurement three times, using different parts of the scale. Why? Take the average of the three readings and plot a curve, using inches as ordinates and the corresponding number of centimeters as abscissæ.

The curve will pass through the origin or zero-point of each scale. Why?

From your curve find the value of 1 inch in centimeters.

What is the true value? What is your per cent of error?

3. A determination of the relation of bending of beam to load gave following results:

Loads.	Deflections.
10 lbs.	0.05 inch.
20 "	0.10 "
30 "	0.15 "
40 "	0.21 "
50 "	0.25 "
60 "	0.29 "
70 "	0.35 "

Plot curve showing relation of deflection to load, using loads as ordinates.

4. Plot a straight line such that it shall gain 3 units of abscissæ for every unit gained as ordinates.

5. A determination of Boyle's Law gave the following data:

Pressure.	Volume of Gas.
82.1	12.03
88.2	11.20
96.2	10.26
105.5	9.35
118.9	8.31
135.5	7.29
160.1	6.17

Plot the curve of above values, using pressures as ordinates.

6. A determination showing the effect of length of a beam upon its stiffness gave the following data:

Length.	Deflection.
10 inches	.005 inch
20 "	.035 "
30 "	.120 "
40 "	.285 "

Plot curve showing relation of deflections to length, using deflections as abscissæ.

7. The area of a circle varies with square of its diameter. Plot a curve to show relation of area to diameter in circle whose successive diameters are 1, 2, 3, 4, 5, and 6. Use diameters as ordinates.

8. What is the equation of the line which cuts the axis of Y at a point 3.4 above the origin and which rises 5 units for every 8 of horizontal distance?

9. If the intercept of a line is .56 and its slope is 2.58, what is its equation?

10. A line has two points whose coördinates are (8.5) and (3.1). Plot the line and obtain its equation.

11. What is the slope of a line if its intercept is 4 and $x=12$ when $y=6.8$?

12. The slope of a certain line is .532, and it passes through the origin. What is the ordinate of a point on this line whose abscissa is 2.3?

13. Write the equation of the following lines. a=intercept and m=slope.

(a) $a=5$,	$m=\ .13$.
(b) $a=-5$,	$m=3$.
(c) $a=-2.3$,	$m=\ .70$.
(d) $a=3$,	$m=-.68$.
(e) $a=-2.5$,	$m=-.45$.

14. Draw a sketch to show the general character of each of the lines described in Problem 13.

SIMPLE TRIGONOMETRIC FUNCTIONS

Let *ABC*, Fig. 212, be any given angle, and let *P* be *any* point on the line *BC*. From *P* draw *PN* perpendicular to the side *BA*, thus forming the right-angled triangle *PNB*. Suppose we measure *BP* and find it to be 10 inches long and *PN* and find it to be 5.05 inches long. Then

$$\frac{PN}{BP}=\frac{5.05}{10}=.505.$$

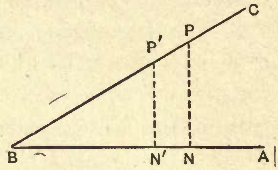

FIG. 212.

Now let us choose another point on *BC*, as *P′*, draw the perpendicular *P′N′* and measure lines *BP′* and *P′N′*. If *BP′* is $\frac{4}{5}$ as long as *BP*, or 8 inches, then *P′N′* will be $\frac{4}{5}$ as long as *PN*, or $5.05\times4=4.04$ inches. The ratio $\frac{P'N'}{BP'}=\frac{4.04}{8}=.505$ or the same as that of *PN* to *BP*. So we might choose any point on *BC* and always obtain the same ratio of the perpendicular to the hypothenuse so long as we use the same angle *ABC*.

Similarly, $\frac{BN}{BP}$ gives a second constant ratio for the given angle, and $\frac{PN}{BN}$ a third, all of which are independent of the position of *P* and dependent only on the angle *ABC*.

If the angle *ABC* be changed, then the ratios $\frac{PN}{BP}$, etc., will have new values, but these again will be the same, no matter where *P* is taken on the line *BC*. Each angle thus has a certain number of constant ratios among which

are the three here given, and these ratios are given distinguishing names.

The ratio $\frac{PN}{BP}$ is called the *sine* of the angle ABC.

The ratio $\frac{BN}{BP}$ is called the *cosine* of the angle ABC

The ratio $\frac{PN}{BN}$ is called the *tangent* of the angle ABC.

Definitions.—Let B be an angle (not the right angle) of a right-angled triangle. The *sine* of the angle B is the ratio of the side *opposite* the angle to the hypothenuse of the triangle.

The *cosine* of the angle B is the ratio of the side *adjacent* the angle to the hypothenuse of the triangle.

The *tangent* of the angle B is the ratio of the side *opposite* to the side *adjacent.*

In calculations, sine, cosine, and tangent are always written for brevity, sin, cos, tan. Thus, $\sin 30° = .500$; $\cos 45° = .707$; $\tan 50° = 1.19$.

The values of these ratios have been calculated for all angles, and are given in what are called tables of trigonometric functions. Such tables, with the values carried out to three decimals, will be found on the following page. See also page 68.

TRIGONOMETRIC FUNCTIONS

A	Sin.	Cos.	Tan.	*A*	Sin.	Cos.	Tan.
0	.000	1.000	.000				
1	.017	.999	.017	46	.719	.695	1.04
2	.035	.999	.035	47	.731	.682	1.07
3	.052	.999	.052	48	.743	.669	1.11
4	.070	.998	.070	49	.755	.656	1.15
5	.087	.996	.087	50	.766	.643	1.19
6	.105	.995	.105	51	.777	.629	1.23
7	.122	.993	.123	52	.788	.616	1.28
8	.139	.990	.141	53	.799	.602	1.33
9	.156	.988	.158	54	.809	.588	1.38
10	.174	.985	.176	55	.819	.574	1.43
11	.191	.982	.194	56	.829	.559	1.48
12	.208	.978	.213	57	.839	.545	1.54
13	.225	.974	.231	58	.848	.530	1.60
14	.242	.970	.249	59	.857	.515	1.66
15	.259	.966	.268	60	.866	.500	1.73
16	.276	.961	.287	61	.875	.485	1.80
17	.292	.956	.306	62	.883	.469	1.88
18	.309	.951	.325	63	.891	.454	1.96
19	.326	.946	.344	64	.898	.438	2.05
20	.342	.940	.364	65	.906	.423	2.14
21	.358	.934	.384	66	.914	.407	2.25
22	.375	.927	.404	67	.921	.391	2.36
23	.391	.921	.424	68	.927	.375	2.48
24	.407	.914	.445	69	.934	.358	2.61
25	.423	.906	.466	70	.940	.342	2.75
26	.438	.898	.488	71	.946	.326	2.90
27	.454	.891	.510	72	.951	.309	3.08
28	.469	.883	.532	73	.956	.292	3.27
29	.485	.875	.554	74	.961	.276	3.49
30	.500	.866	.577	75	.966	.259	3.73
31	.515	.857	.601	76	.970	.242	4.01
32	.530	.848	.625	77	.974	.225	4.33
33	.545	.839	.649	78	.978	.208	4.70
34	.559	.829	.675	79	.982	.191	5.14
35	.574	.819	.700	80	.985	.174	5.67
36	.588	.809	.727	81	.988	.156	6.31
37	.602	.799	.754	82	.990	.139	7.12
38	.616	.788	.781	83	.993	.122	8.14
39	.629	.777	.810	84	.995	.105	9.51
40	.643	.766	.839	85	.996	.087	11.43
41	.656	.755	.869	86	.998	.070	14.30
42	.669	.743	.900	87	.999	.052	19.08
43	.682	.731	.933	88	.999	.035	28.64
44	.695	.719	.966	89	.999	.017	57.28
45	.707	.707	1.000	90	1.000	.000	Infinity

INDEX

A

B

C

D

E

F

G